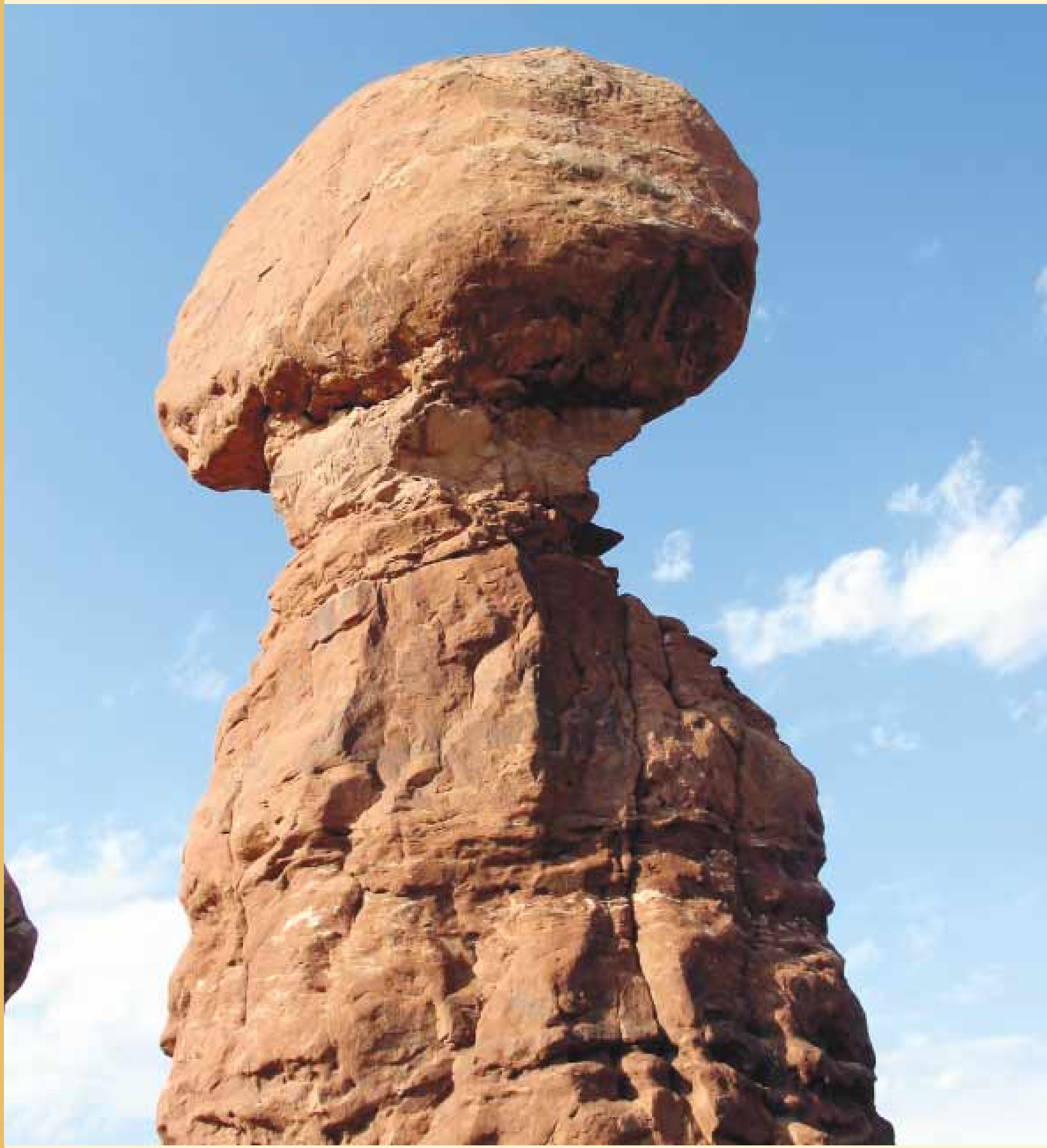
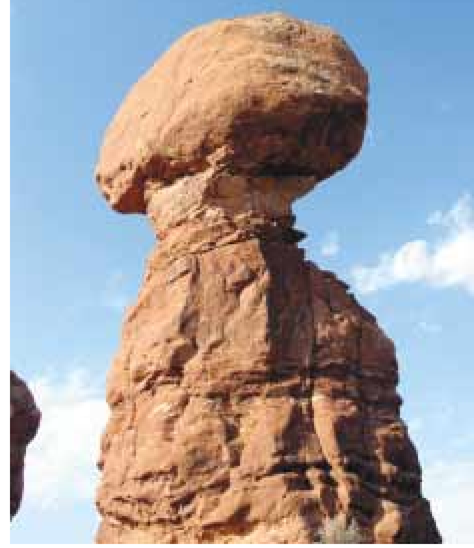
Chapter 12

Static Equilibrium and Elasticity

CHAPTER OUTLINE



▲ *Balanced Rock in Arches National Park, Utah, is a 3 000 000-kg boulder that has been in stable equilibrium for several millennia. It had a smaller companion nearby, called "Chip Off the Old Block," which fell during the winter of 1975. Balanced Rock appeared in an early scene of the movie* Indiana Jones and the Last Crusade. *We will study the conditions under which an object is in equilibrium in this chapter. (John W. Jewett, Jr.)*



In Chapters 10 and 11 we studied the dynamics of rigid objects. Part of this current chapter addresses the conditions under which a rigid object is in equilibrium. The term *equilibrium* implies either that the object is at rest or that its center of mass moves with constant velocity relative to the observer. We deal here only with the former case, in which the object is in *static equilibrium.* Static equilibrium represents a common situation in engineering practice, and the principles it involves are of special interest to civil engineers, architects, and mechanical engineers. If you are an engineering student, you will undoubtedly take an advanced course in statics in the future.

The last section of this chapter deals with how objects deform under load conditions. An *elastic* object returns to its original shape when the deforming forces are removed. Several elastic constants are defined, each corresponding to a different type of deformation.

12.1 The Conditions for Equilibrium

In Chapter 5 we found that one necessary condition for equilibrium is that the net force acting on an object must be zero. If the object is modeled as a particle, then this is the only condition that must be satisfied for equilibrium. The situation with real (extended) objects is more complex, however, because these objects often cannot be modeled as particles. For an extended object to be in static equilibrium, a second condition must be satisfied. This second condition involves the net torque acting on the extended object.

Consider a single force **F** acting on a rigid object, as shown in Figure 12.1. The effect of the force depends on the location of its point of application *P*. If **r** is the position vector of this point relative to *O*, the torque associated with the force **F** about *O* is given by Equation 11.1:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

Recall from the discussion of the vector product in Section 11.1 that the vector $\boldsymbol{\tau}$ is perpendicular to the plane formed by **r** and **F**. You can use the right-hand rule to determine the direction of $\boldsymbol{\tau}$ as shown in Figure 11.2. Hence, in Figure 12.1 $\boldsymbol{\tau}$ is directed toward you out of the page.

As you can see from Figure 12.1, the tendency of **F** to rotate the object about an axis through *O* depends on the moment arm *d*, as well as on the magnitude of **F**. Recall that the magnitude of $\boldsymbol{\tau}$ is Fd (see Eq. 10.19). According to Equation 10.21, the net torque on a rigid object will cause it to undergo an angular acceleration.

In the current discussion, we want to look at those rotational situations in which the angular acceleration of a rigid object is zero. Such an object is in **rotational equilibrium**. Because $\Sigma\boldsymbol{\tau} = I\alpha$ for rotation about a fixed axis, the necessary condition for rotational equilibrium is that **the net torque about any axis must be zero**. We now

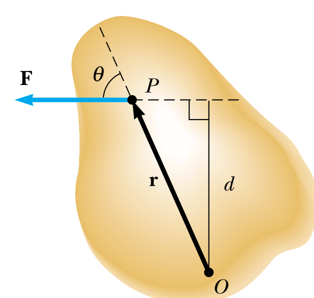


Figure 12.1 A single force **F** acts on a rigid object at the point *P*.

have two necessary conditions for equilibrium of an object:

1. The resultant external force must equal zero:

$$\sum \mathbf{F} = 0 \tag{12.1}$$

2. The resultant external torque about *any* axis must be zero:

$$\sum \boldsymbol{\tau} = 0 \tag{12.2}$$

The first condition is a statement of translational equilibrium; it tells us that the linear acceleration of the center of mass of the object must be zero when viewed from an inertial reference frame. The second condition is a statement of rotational equilibrium and tells us that the angular acceleration about any axis must be zero. In the special case of **static equilibrium**, which is the main subject of this chapter, the object is at rest relative to the observer and so has no linear or angular speed (that is, $v_{CM} = 0$ and $\omega = 0$).

PITFALL PREVENTION

12.1 Zero Torque

Zero net torque does not mean an absence of rotational motion. An object which is rotating at a constant angular speed can be under the influence of a net torque of zero. This is analogous to the translational situation—zero net force does not mean an absence of translational motion.

Quick Quiz 12.1 Consider the object subject to the two forces in Figure 12.2. Choose the correct statement with regard to this situation. (a) The object is in force equilibrium but not torque equilibrium. (b) The object is in torque equilibrium but not force equilibrium. (c) The object is in both force and torque equilibrium. (d) The object is in neither force nor torque equilibrium.

Quick Quiz 12.2 Consider the object subject to the three forces in Figure 12.3. Choose the correct statement with regard to this situation. (a) The object is in force equilibrium but not torque equilibrium. (b) The object is in torque equilibrium but not force equilibrium. (c) The object is in both force and torque equilibrium. (d) The object is in neither force nor torque equilibrium.

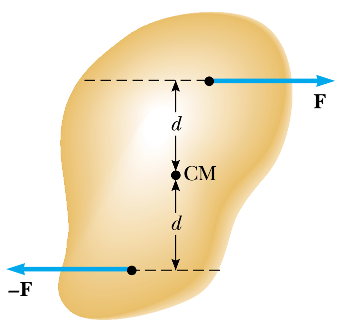


Figure 12.2 (Quick Quiz 12.1) Two forces of equal magnitude are applied at equal distances from the center of mass of a rigid object.

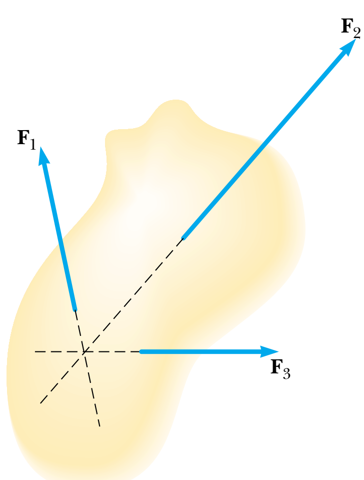


Figure 12.3 (Quick Quiz 12.2) Three forces act on an object. Notice that the lines of action of all three forces pass through a common point.

The two vector expressions given by Equations 12.1 and 12.2 are equivalent, in general, to six scalar equations: three from the first condition for equilibrium, and three from the second (corresponding to x, y, and z components). Hence, in a complex system involving several forces acting in various directions, you could be faced with solving a set of equations with many unknowns. Here, we restrict our discussion to situations in which all the forces lie in the xy plane. (Forces whose vector representations are in the same plane are said to be *coplanar.*) With this restriction, we must deal with only three scalar equations. Two of these come from balancing the forces in the x and y directions. The third comes from the torque equation—namely, that the net torque about a perpendicular axis through *any* point in the xy plane must be zero. Hence, the two conditions of equilibrium provide the equations

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau_z = 0 \tag{12.3}$$

where the location of the axis of the torque equation is arbitrary, as we now show.

Regardless of the number of forces that are acting, if an object is in translational equilibrium and if the net torque is zero about one axis, then the net torque must also be zero about any other axis. The axis can pass through a point that is inside or outside the boundaries of the object. Consider an object being acted on by several forces such that the resultant force $\Sigma \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = 0$. Figure 12.4 describes this situation (for clarity, only four forces are shown). The point of application of $\mathbf{F}_1$ relative to O is specified by the position vector $\mathbf{r}_1$. Similarly, the points of application of $\mathbf{F}_2$, $\mathbf{F}_3$, . . . are specified by $\mathbf{r}_2$, $\mathbf{r}_3$, . . . (not shown). The net torque about an axis through O is

$$\sum \boldsymbol{\tau}_O = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3 + \cdots$$

Now consider another arbitrary point O' having a position vector $\mathbf{r}'$ relative to O. The point of application of $\mathbf{F}_1$ relative to O' is identified by the vector $\mathbf{r}_1 - \mathbf{r}'$. Likewise, the point of application of $\mathbf{F}_2$ relative to O' is $\mathbf{r}_2 - \mathbf{r}'$, and so forth. Therefore, the torque about an axis through O' is

$$\sum \boldsymbol{\tau}_{O'} = (\mathbf{r}_1 - \mathbf{r}') \times \mathbf{F}_1 + (\mathbf{r}_2 - \mathbf{r}') \times \mathbf{F}_2 + (\mathbf{r}_3 - \mathbf{r}') \times (\mathbf{F}_3 + \cdots$$
$$= \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3 + \cdots - \mathbf{r}' \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots)$$

Because the net force is assumed to be zero (given that the object is in translational equilibrium), the last term vanishes, and we see that the torque about an axis through O' is equal to the torque about an axis through O. Hence, **if an object is in translational equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis.**

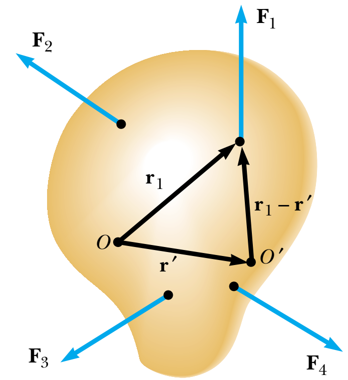


Figure 12.4 Construction showing that if the net torque is zero about origin O, it is also zero about any other origin, such as O'.

12.2 More on the Center of Gravity

We have seen that the point at which a force is applied can be critical in determining how an object responds to that force. For example, two equal-magnitude but oppositely directed forces result in equilibrium if they are applied at the same point on an object. However, if the point of application of one of the forces is moved, so that the two forces no longer act along the same line of action, then the object undergoes an angular acceleration.

Whenever we deal with a rigid object, one of the forces we must consider is the gravitational force acting on it, and we must know the point of application of this force. As we learned in Section 9.5, associated with every object is a special point called its center of gravity. All the various gravitational forces acting on all the various mass elements of the object are equivalent to a single gravitational force acting through this point. Thus, to compute the torque due to the gravitational force on an object of mass M, we need only consider the force $M\mathbf{g}$ acting at the center of gravity of the object.

How do we find this special point? As we mentioned in Section 9.5, if we assume that $\mathbf{g}$ is uniform over the object, then the center of gravity of the object coincides with its center of mass. To see that this is so, consider an object of arbitrary shape lying in the xy plane, as illustrated in Figure 12.5. Suppose the object is divided into a large number of particles of masses m_1, m_2, m_3, . . . having coordinates (x_1, y_1), (x_2, y_2), (x_3, y_3), ... In Equation 9.28 we defined the x coordinate of the center of mass of such an object to be

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_ix_i}{\sum_i m_i}$$

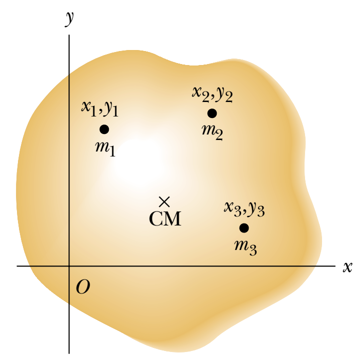


Figure 12.5 An object can be divided into many small particles each having a specific mass and specific coordinates. These particles can be used to locate the center of mass.

We use a similar equation to define the y coordinate of the center of mass, replacing each x with its y counterpart.

Let us now examine the situation from another point of view by considering the gravitational force exerted on each particle, as shown in Figure 12.6. Each particle contributes a torque about the origin equal in magnitude to the particle's weight mg multiplied by its moment arm. For example, the magnitude of the torque due to the force $m_1\mathbf{g}_1$ is $m_1g_1x_1$, where g_1 is the value of the gravitational acceleration at the position of the particle of mass m_1. We wish to locate the center of gravity, the point at which application of the single gravitational force $M\mathbf{g}$ (where $M = m_1 + m_2 + m_3 + \cdots$ is the total mass of the object) has the same effect on rotation as does the combined effect of all the individual gravitational forces $m_i\mathbf{g}_i$. Equating the torque resulting from $M\mathbf{g}$ acting at the center of gravity to the sum of the torques acting on the individual particles gives

$$(m_1g_1 + m_2g_2 + m_3g_3 + \cdots)x_{\text{CG}} = m_1g_1x_1 + m_2g_2x_2 + m_3g_3x_3 + \cdots$$

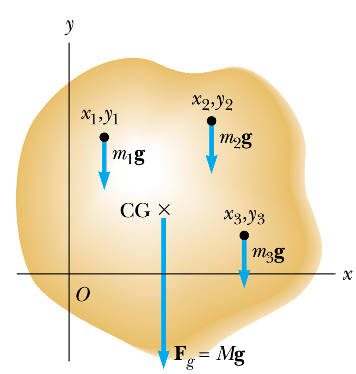


Figure 12.6 The center of gravity of an object is located at the center of mass if $\mathbf{g}$ is constant over the object.

This expression accounts for the fact that the value of g can in general vary over the object. If we assume uniform g over the object (as is usually the case), then the

g terms cancel and we obtain

$$x_{CG} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \tag{12.4}$$

Comparing this result with Equation 9.28, we see that **the center of gravity is located at the center of mass as long as g is uniform over the entire object**. In several examples presented in the next section, we will deal with homogeneous, symmetric objects. The center of gravity for any such object coincides with its geometric center.

Quick Quiz 12.3 A meter stick is supported on a fulcrum at the 25-cm mark. A 0.50-kg object is hung from the zero end of the meter stick, and the stick is balanced horizontally. The mass of the meter stick is (a) 0.25 kg (b) 0.50 kg (c) 0.75 kg (d) 1.0 kg (e) 2.0 kg (f) impossible to determine.

12.3 Examples of Rigid Objects in Static Equilibrium


Charles D. Winters

Figure 12.7 This one-bottle wine holder is a surprising display of static equilibrium. The center of gravity of the system (bottle plus holder) is directly over the support point.

The photograph of the one-bottle wine holder in Figure 12.7 shows one example of a balanced mechanical system that seems to defy gravity. For the system (wine holder plus bottle) to be in equilibrium, the net external force must be zero (see Eq. 12.1) and the net external torque must be zero (see Eq. 12.2). The second condition can be satisfied only when the center of gravity of the system is directly over the support point.

When working static equilibrium problems, you must recognize all the external forces acting on the object. Failure to do so results in an incorrect analysis. When analyzing an object in equilibrium under the action of several external forces, use the following procedure.

PROBLEM-SOLVING STRATEGY

OBJECTS IN STATIC EQUILIBRIUM

- Draw a simple, neat diagram of the system.
- Isolate the object being analyzed. Draw a free-body diagram. Then show and label all external forces acting on the object, indicating where those forces are applied. Do not include forces exerted by the object on its surroundings. (For systems that contain more than one object, draw a *separate* free-body diagram for each one.) Try to guess the correct direction for each force.
- Establish a convenient coordinate system and find the components of the forces on the object along the two axes. Then apply the first condition for equilibrium. Remember to keep track of the signs of the various force components.
- Choose a convenient axis for calculating the net torque on the object. Remember that the choice of origin for the torque equation is arbitrary; therefore choose an origin that simplifies your calculation as much as possible. Note that a force that acts along a line passing through the point chosen as the origin gives zero contribution to the torque and so can be ignored.
- The first and second conditions for equilibrium give a set of linear equations containing several unknowns, and these equations can be solved simultaneously. If the direction you selected for a force leads to a negative value, do not be alarmed; this merely means that the direction of the force is the opposite of what you guessed.

Example 12.1 The Seesaw Revisited

A seesaw consisting of a uniform board of mass M and length ℓ supports a father and daughter with masses m_f and m_d, respectively, as shown in Figure 12.8. The support (called the *fulcrum*) is under the center of gravity of the board, the father is a distance d from the center, and the daughter is a distance $\ell/2$ from the center.

(A) Determine the magnitude of the upward force **n** exerted by the support on the board.

Solution First note that, in addition to **n**, the external forces acting on the board are the downward forces exerted by each person and the gravitational force acting on the board. We know that the board's center of gravity is at its geometric center because we are told that the board is uniform. Because the system is in static equilibrium, the net force on the board is zero. Thus, the upward force **n** must balance all the downward forces. From $\Sigma F_y = 0$, and defining upward as the positive y direction, we have

$$n - m_f g - m_d g - Mg = 0$$

$$n = \boxed{m_f g + m_d g + Mg}$$

(The equation $\Sigma F_x = 0$ also applies, but we do not need to consider it because no forces act horizontally on the board.)

(B) Determine where the father should sit to balance the system.

Solution To find this position, we must invoke the second condition for equilibrium. If we take an axis perpendicular to the page through the center of gravity of the board as the axis for our torque equation, the torques produced by **n** and the gravitational force acting on the board are zero. We see from $\Sigma\tau = 0$ that

$$(m_f g)(d) - (m_d g)\frac{\ell}{2} = 0$$

$$d = \boxed{\left(\frac{m_d}{m_f}\right)\tfrac{1}{2}\ell}$$

This is the same result that we obtained in Example 11.6 by evaluating the angular acceleration of the system and setting the angular acceleration equal to zero.

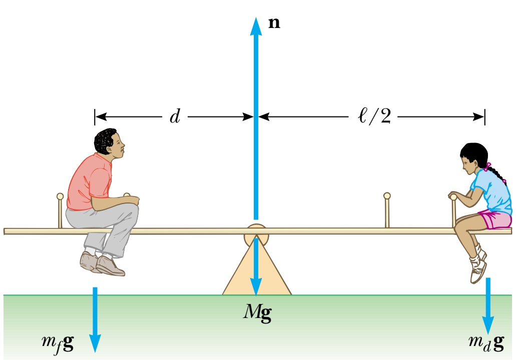


Figure 12.8 (Example 12.1) A balanced system.

What If? Suppose we had chosen another point through which the rotation axis were to pass. For example, suppose the axis is perpendicular to the page and passes through the location of the father. Does this change the results to parts (A) and (B)?

Answer Part (A) is unaffected because the calculation of the net force does not involve a rotation axis. In part (B), we would conceptually expect there to be no change if a different rotation axis is chosen because the second condition of equilibrium claims that the torque is zero about any rotation axis.

Let us verify this mathematically. Recall that the sign of the torque associated with a force is positive if that force tends to rotate the system counterclockwise, while the sign of the torque is negative if the force tends to rotate the system clockwise. In the case of a rotation axis passing through the location of the father, $\Sigma\tau = 0$ yields

$$n(d) - (Mg)(d) - (m_d g)(d + \ell/2) = 0$$

From part (A) we know that $n = m_f g + m_d g + Mg$. Thus, we can substitute this expression for n and solve for d:

$$(m_f g + m_d g + Mg)(d) - (Mg)(d) - (m_d g)\left(d + \frac{\ell}{2}\right) = 0$$

$$(m_f g)(d) - (m_d g)\left(\frac{\ell}{2}\right) = 0$$

$$d = \left(\frac{m_d}{m_f}\right)\tfrac{1}{2}\ell$$

This result is in agreement with the one we obtained in part (B).

Example 12.2 A Weighted Hand

A person holds a 50.0-N sphere in his hand. The forearm is horizontal, as shown in Figure 12.9a. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0 cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of the forearm.

Solution We simplify the situation by modeling the forearm as a bar as shown in Figure 12.9b, where **F** is the upward force exerted by the biceps and **R** is the downward force exerted by the upper arm at the joint. From the first condition for equilibrium, we have, with upward as the positive y direction,

$$(1) \qquad \sum F_y = F - R - 50.0\ \text{N} = 0$$

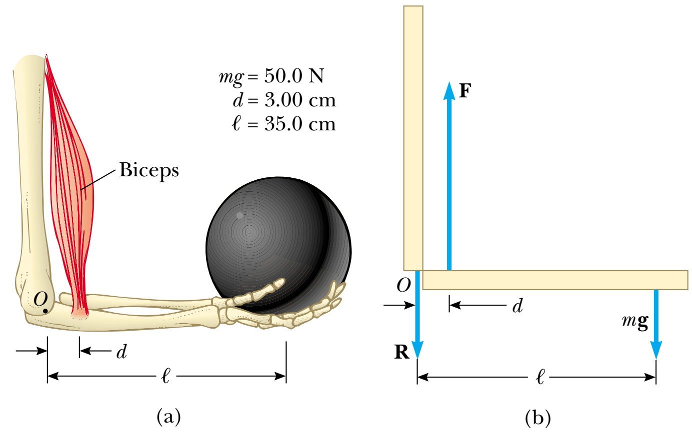


Figure 12.9 (Example 12.2) (a) The biceps muscle pulls upward with a force **F** that is essentially at a right angle to the forearm. (b) The mechanical model for the system described in part (a).

From the second condition for equilibrium, we know that the sum of the torques about any point must be zero. With the joint O as the axis, we have

$$\sum \tau = Fd - mg\ell = 0$$

$$F(3.00 \text{ cm}) - (50.0 \text{ N})(35.0 \text{ cm}) = 0$$

$$F = \boxed{583 \text{ N}}$$

This value for F can be substituted into Equation (1) to give $R = 533$ N. As this example shows, the forces at joints and in muscles can be extremely large.

Example 12.3 Standing on a Horizontal Beam

Interactive

A uniform horizontal beam with a length of 8.00 m and a weight of 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 53.0° with the beam (Fig. 12.10a). If a 600-N person stands 2.00 m from the wall, find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.

Solution Conceptualize this problem by imagining that the person in Figure 12.10 moves outward on the beam. It seems reasonable that the farther he moves outward, the larger the torque that he applies about the pivot and the larger the tension in the cable must be to balance this torque. Because the system is at rest, we categorize this as a static equilibrium problem. We begin to analyze the problem by identifying all the external forces acting on the beam: the 200-N gravitational force, the force **T** exerted by the cable, the force **R** exerted by the wall at the pivot, and the 600-N force that the person exerts on the beam. These forces are all indicated in the free-body diagram for the beam shown in Figure 12.10b. When we assign directions for forces, it is sometimes helpful to imagine what would happen if a force were suddenly removed. For example, if the wall were to vanish suddenly, the left end of the beam would move to the left as it begins to fall. This tells us that the wall is not only holding the beam up but is also pressing outward against it. Thus, we draw the vector **R** as shown in Figure 12.10b. If we resolve **T** and **R** into horizontal and vertical components, as shown in Figure 12.10c, and apply the first condition for equilibrium, we obtain

(1) $$\sum F_x = R\cos\theta - T\cos 53.0° = 0$$

(2) $$\sum F_y = R\sin\theta + T\sin 53.0° - 600 \text{ N} - 200 \text{ N} = 0$$

where we have chosen rightward and upward as our positive directions. Because R, T, and θ are all unknown, we cannot obtain a solution from these expressions alone. (The number of simultaneous equations must equal the number of unknowns for us to be able to solve for the unknowns.)

Now let us invoke the condition for rotational equilibrium. A convenient axis to choose for our torque equation is the one that passes through the pin connection. The feature that makes this point so convenient is that the force **R** and the horizontal component of **T** both have a moment arm of zero; hence, these forces provide no torque about this point. Recalling our counterclockwise-equals-positive convention for the sign of the torque about an axis and noting that the moment arms of the 600-N, 200-N, and $T\sin 53.0°$ forces are 2.00 m, 4.00 m, and 8.00 m, respectively, we obtain

(3) $$\sum \tau = (T\sin 53.0°)(8.00 \text{ m}) - (600 \text{ N})(2.00 \text{ m}) - (200 \text{ N})(4.00 \text{ m}) = 0$$

$$T = \boxed{313 \text{ N}}$$

Thus, the torque equation with this axis gives us one of the unknowns directly! We now substitute this value into Equations (1) and (2) and find that

$$R\cos\theta = 188 \text{ N}$$

$$R\sin\theta = 550 \text{ N}$$

We divide the second equation by the first and, recalling the trigonometric identity $\sin\theta/\cos\theta = \tan\theta$, we obtain

$$\tan\theta = \frac{550 \text{ N}}{188 \text{ N}} = 2.93$$

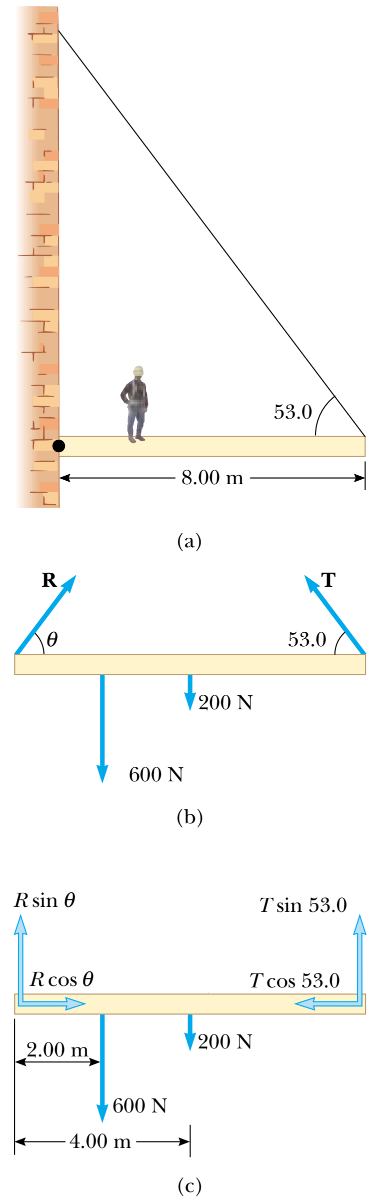


Figure 12.10 (Example 12.3) (a) A uniform beam supported by a cable. A person walks outward on the beam. (b) The free-body diagram for the beam. (c) The free-body diagram for the beam showing the components of **R** and **T**.

$$\theta = \boxed{71.1^\circ}$$

This positive value indicates that our estimate of the direction of **R** was accurate.

Finally,

$$R = \frac{188\text{ N}}{\cos\theta} = \frac{188\text{ N}}{\cos 71.1^\circ} = \boxed{580\text{ N}}$$

To finalize this problem, note that if we had selected some other axis for the torque equation, the solution might differ in the details, but the answers would be the same. For example, if we had chosen an axis through the center of gravity of the beam, the torque equation would involve both T and R. However, this equation, coupled with Equations (1) and (2), could still be solved for the unknowns. Try it!

When many forces are involved in a problem of this nature, it is convenient in your analysis to set up a table. For instance, for the example just given, we could construct the following table. Setting the sum of the terms in the last column equal to zero represents the condition of rotational equilibrium.

Force component	Moment arm relative to O (m)	Torque about O (N·m)
$T\sin 53.0^\circ$	8.00	$(8.00)\,T\sin 53.0^\circ$
$T\cos 53.0^\circ$	0	0
200 N	4.00	$-(4.00)(200)$
600 N	2.00	$-(2.00)(600)$
$R\sin\theta$	0	0
$R\cos\theta$	0	0

What If? **What if the person walks farther out on the beam? Does T change? Does R change? Does θ change?**

Answer T must increase because the weight of the person exerts a larger torque about the pin connection, which must be countered by a larger torque in the opposite direction due to an increased value of T. If T increases, the vertical component of **R** decreases to maintain force equilibrium in the vertical direction. But force equilibrium in the horizontal direction requires an increased horizontal component of **R** to balance the horizontal component of the increased **T**. This suggests that θ will become smaller, but it is hard to predict what will happen to R. Problem 26 allows you to explore the behavior of R.

At the Interactive Worked Example link at **http://www.pse6.com,** *you can adjust the position of the person and observe the effect on the forces.*

Example 12.4 The Leaning Ladder

Interactive

A uniform ladder of length ℓ rests against a smooth, vertical wall (Fig. 12.11a). If the mass of the ladder is m and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$, find the minimum angle θ_{min} at which the ladder does not slip.

Solution The free-body diagram showing all the external forces acting on the ladder is illustrated in Figure 12.11b. The force exerted by the ground on the ladder is the vector sum of a normal force **n** and the force of static friction $\mathbf{f}_s$. The reaction force **P** exerted by the wall on the ladder is horizontal because the wall is frictionless. Notice how we have included only forces that act on the ladder. For example, the forces exerted by the ladder on the ground and on the wall are not part of the problem and thus do not appear in the free-body diagram. Applying the first condition for equilibrium to the ladder, we have

$$(1) \qquad \sum F_x = f_s - P = 0$$

$$(2) \qquad \sum F_y = n - mg = 0$$

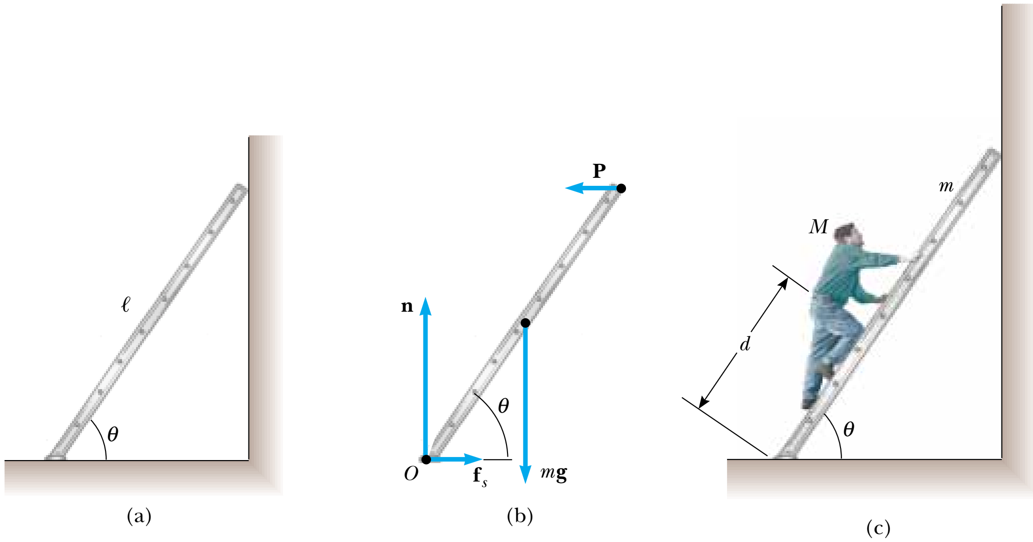


Figure 12.11 (Example 12.4) (a) A uniform ladder at rest, leaning against a smooth wall. The ground is rough. (b) The free-body diagram for the ladder. (c) A person of mass M begins to climb the ladder when it is at the minimum angle found in part (a) of the example. Will the ladder slip?

The first equation tells us that $P = f_s$. From the second equation we see that $n = mg$. Furthermore, when the ladder is on the verge of slipping, the force of friction must be a maximum, which is given by $f_{s,\,\max} = \mu_s n$. (Recall Eq. 5.8: $f_s \le \mu_s n$.) Thus, we must have $P = f_s = \mu_s n = \mu_s mg$.

To find $\theta_{\min}$, we must use the second condition for equilibrium. When we take the torques about an axis through the origin O at the bottom of the ladder, we have

$$(3) \qquad \sum \tau_O = P\ell \sin\theta - mg\frac{\ell}{2}\cos\theta = 0$$

This expression gives

$$\tan\theta_{\min} = \frac{mg}{2P} = \frac{mg}{2\mu_s mg} = \frac{1}{2\mu_s} = 1.25$$

$$\theta_{\min} = \boxed{51^\circ}$$

What If? **What if a person begins to climb the ladder when the angle is 51°? Will the presence of a person on the ladder make it more or less likely to slip?**

Answer The presence of the additional weight of a person on the ladder will increase the clockwise torque about its base in Figure 12.11b. To maintain static equilibrium, the counterclockwise torque must increase, which can occur if P increases. Because equilibrium in the horizontal direction tells us that $P = f_s$, this would suggest that the friction force rises above the maximum value $f_{s,\max}$ and the ladder slips. However, the increased weight of the person also causes n to increase, which increases the maximum friction force $f_{s,\max}$! Thus, it is not clear conceptually whether the ladder is more or less likely to slip.

Imagine that the person of mass M is at a position d that is measured along the ladder from its base (Fig. 12.11c). Equations (1) and (2) can be rewritten

$$(4) \qquad \sum F_x = f_s - P = 0$$

$$(5) \qquad \sum F_y = n - (m + M)g = 0$$

Equation (3) can be rewritten

$$\sum \tau_O = P\ell \sin\theta - mg\frac{\ell}{2}\cos\theta - Mgd\cos\theta = 0$$

Solving this equation for $\tan\theta$, we find

$$\tan\theta = \frac{mg(\ell/2) + Mgd}{P\ell}$$

Incorporating Equations (4) and (5), and imposing the condition that the ladder is about to slip, this becomes

$$(6) \qquad \tan\theta_{\min} = \frac{m(\ell/2) + Md}{\mu_s \ell(m + M)}$$

When the person is at the bottom of the ladder, $d = 0$. In this case, there is no additional torque about the bottom of the ladder and the increased normal force causes the maximum static friction force to increase. Thus, the ladder is less likely to slip than in the absence of the person. As the person climbs and d becomes larger, however, the numerator in Equation (6) becomes larger. Thus, the minimum angle at which the ladder does not slip increases. Eventually, as the person climbs higher, the minimum angle becomes larger than 51° and the ladder slips. The particular value of d at which the ladder slips depends on the coefficient of friction and the masses of the person and the ladder.

At the Interactive Worked Example link at **http://www.pse6.com,** *you can adjust the angle of the ladder and watch what happens when it is released.*

Example 12.5 Negotiating a Curb

(A) Estimate the magnitude of the force **F** a person must apply to a wheelchair's main wheel to roll up over a sidewalk curb (Fig. 12.12a). This main wheel that comes in contact with the curb has a radius r, and the height of the curb is h.

Solution Normally, the person's hands supply the required force to a slightly smaller wheel that is concentric with the main wheel. For simplicity, we assume that the radius of the smaller wheel is the same as the radius of the main wheel. Let us estimate a combined weight of $mg = 1\,400$ N for the person and the wheelchair and choose a wheel radius of $r = 30$ cm. We also pick a curb height of $h = 10$ cm. We assume that the wheelchair and occupant are symmetric, and that each wheel supports a weight of 700 N. We then proceed to analyze only one of the wheels. Figure 12.12b shows the geometry for a single wheel.

When the wheel is just about to be raised from the street, the reaction force exerted by the ground on the wheel at point B goes to zero. Hence, at this time only three forces act on the wheel, as shown in the free-body diagram in Figure 12.12c. However, the force **R**, which is the force exerted by the curb on the wheel, acts at point A, and so if we choose to have our axis of rotation pass through point A, we do not need to include **R** in our torque equation. From the triangle OAC shown in Figure 12.12b, we see that the moment arm d of the gravitational force $m\mathbf{g}$ acting on the wheel relative to point A is

$$d = \sqrt{r^2 - (r-h)^2} = \sqrt{2rh - h^2}$$

The moment arm of **F** relative to point A is $2r - h$ (see Figure 12.12c). Therefore, the net torque acting on the wheel about point A is

$$mgd - F(2r - h) = 0$$

$$mg\sqrt{2rh - h^2} - F(2r - h) = 0$$

$$F = \frac{mg\sqrt{2rh - h^2}}{2r - h}$$

$$= \frac{(700\ \text{N})\sqrt{2(0.3\ \text{m})(0.1\ \text{m}) - (0.1\ \text{m})^2}}{2(0.3\ \text{m}) - 0.1\ \text{m}}$$

$$= \boxed{3 \times 10^2\ \text{N}}$$

(Notice that we have kept only one digit as significant.) This result indicates that the force that must be applied to each wheel is substantial. You may want to estimate the force required to roll a wheelchair up a typical sidewalk accessibility ramp for comparison.

(B) Determine the magnitude and direction of **R**.

Solution We use the first condition for equilibrium to determine the direction:

$$\sum F_x = F - R\cos\theta = 0$$

$$\sum F_y = R\sin\theta - mg = 0$$

Dividing the second equation by the first gives

$$\tan\theta = \frac{mg}{F} = \frac{700\ \text{N}}{300\ \text{N}}$$

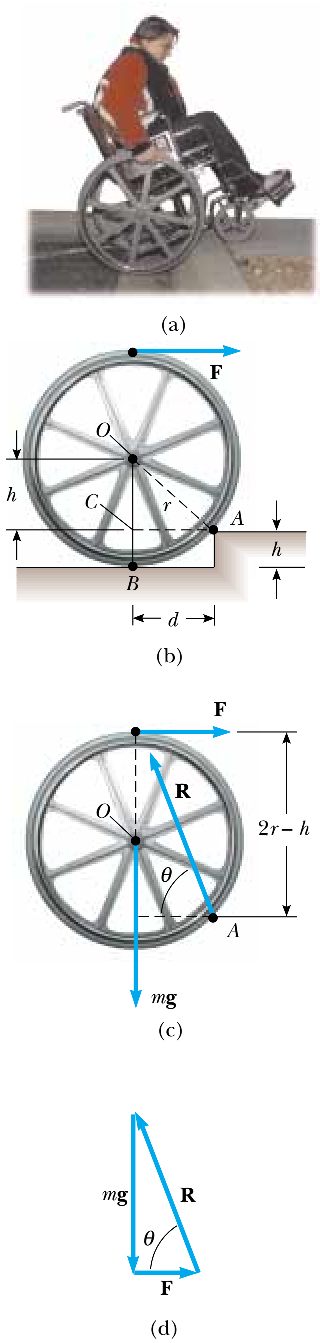


Figure 12.12 (Example 12.5) (a) A person in a wheelchair attempts to roll up over a curb. (b) Details of the wheel and curb. The person applies a force **F** to the top of the wheel. (c) The free-body diagram for the wheel when it is just about to be raised. Three forces act on the wheel at this instant: **F**, which is exerted by the hand; **R**, which is exerted by the curb; and the gravitational force $m\mathbf{g}$. (d) The vector sum of the three external forces acting on the wheel is zero.

$$\boxed{\theta = 70°}$$

We can use the right triangle shown in Figure 12.12d to obtain R:

$$R = \sqrt{(mg)^2 + F^2} = \sqrt{(700\ \text{N})^2 + (300\ \text{N})^2} = \boxed{800\ \text{N}}$$

Application Analysis of a Truss

Roofs, bridges, and other structures that must be both strong and lightweight often are made of trusses similar to the one shown in Figure 12.13a. Imagine that this truss structure represents part of a bridge. To approach this problem, we assume that the structural components are connected by pin joints. We also assume that the entire structure is free to slide horizontally because it rests on "rockers" on each end, which allow it to move back and forth as it undergoes thermal expansion and contraction. We assume the mass of the bridge structure is negligible compared with the load. In this situation, the force exerted by each of the bars (struts) on the hinge pins is a force of tension or of compression and must be along the length of the bar. Let us calculate the force in each strut when the bridge is supporting a 7 200-N load at its center. We will do this by determining the forces that act at the pins.

The force notation that we use here is not of our usual format. Until now, we have used the notation F_{AB} to mean "the force exerted by A on B." For this application, however, the first letter in a double-letter subscript on F indicates the location of the pin on which the force is exerted. The combination of two letters identifies the strut exerting the force on the pin. For example, in Figure 12.13b, F_{AB} is the force exerted by strut AB on the pin at A. The subscripts are symmetric in that strut AB is the same as strut BA and $F_{AB} = F_{BA}$.

First, we apply Newton's second law to the truss as a whole in the vertical direction. Internal forces do not enter into this accounting. We balance the weight of the load with the normal forces exerted at the two ends by the supports on which the bridge rests:

$$\sum F_y = n_A + n_E - F_g = 0$$

$$n_A + n_E = 7\,200 \text{ N}$$

Next, we calculate the torque about A, noting that the overall length of the bridge structure is $L = 50$ m:

$$\sum \tau = Ln_E - (L/2)\,F_g = 0$$

$$n_E = F_g/2 = 3\,600 \text{ N}$$

Although we could repeat the torque calculation for the right end (point E), it should be clear from symmetry arguments that $n_A = 3\,600$ N.

Now let us balance the vertical forces acting on the pin at point A. If we assume that strut AB is in compression, then the force F_{AB} that the strut exerts on the pin at point A has a negative y component. (If the strut is actually in tension, our calculations will result in a negative value for the magnitude of the force, still of the correct size):

$$\sum F_y = n_A - F_{AB} \sin 30° = 0$$

$$F_{AB} = 7\,200 \text{ N}$$

The positive result shows that our assumption of compression was correct.

We can now find the force F_{AC} by considering the horizontal forces acting on the pin at point A. Because point A is not accelerating, we can safely assume that F_{AC} must point toward the right (Fig. 12.13b); this indicates that the bar between points A and C is under tension:

$$\sum F_x = F_{AC} - F_{AB} \cos 30° = 0$$

$$F_{AC} = (7\,200 \text{ N}) \cos 30° = 6\,200 \text{ N}$$

Now consider the vertical forces acting on the pin at point C. We shall assume that strut CB is in tension. (Imagine the subsequent motion of the pin at point C if strut CB were to

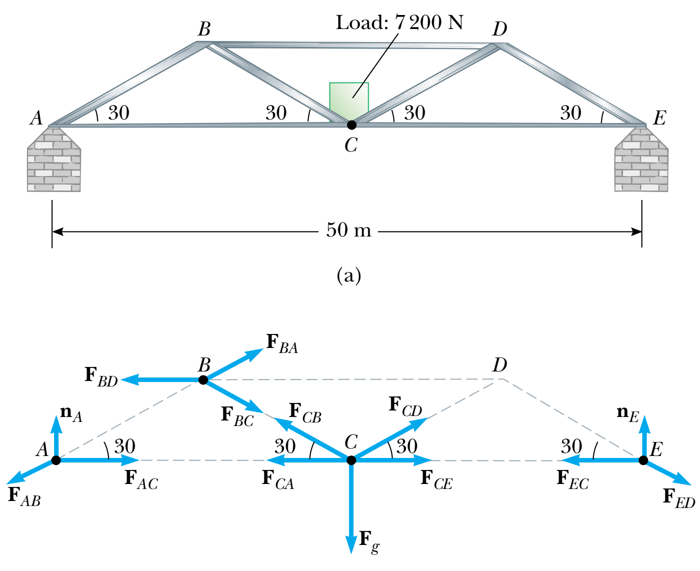


Figure 12.13 (a) Truss structure for a bridge. (b) The forces acting on the pins at points A, B, C, and E. Force vectors are not to scale.

break suddenly.) On the basis of symmetry, we assert that $F_{CB} = F_{CD}$ and $F_{CA} = F_{CE}$:

$$\sum F_y = 2\,F_{CB} \sin 30° - 7\,200\text{ N} = 0$$

$$F_{CB} = 7\,200\text{ N}$$

Finally, we balance the horizontal forces on B, assuming that strut BD is in compression:

$$\sum F_x = F_{BA} \cos 30° + F_{BC} \cos 30° - F_{BD} = 0$$

$$(7\,200\text{ N}) \cos 30° + (7\,200\text{ N}) \cos 30° - F_{BD} = 0$$

$$F_{BD} = 12\,000\text{ N}$$

Thus, the top bar in a bridge of this design must be very strong.

12.4 Elastic Properties of Solids

Except for our discussion about springs in earlier chapters, we have assumed that objects remain rigid when external forces act on them. In reality, all objects are deformable. That is, it is possible to change the shape or the size (or both) of an object by applying external forces. As these changes take place, however, internal forces in the object resist the deformation.

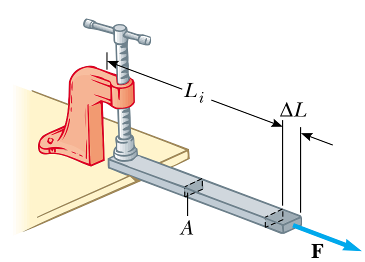
We shall discuss the deformation of solids in terms of the concepts of *stress* and *strain*. **Stress** is a quantity that is proportional to the force causing a deformation; more specifically, stress is the external force acting on an object per unit cross-sectional area. The result of a stress is **strain**, which is a measure of the degree of deformation. It is found that, for sufficiently small stresses, **strain is proportional to stress;** the constant of proportionality depends on the material being deformed and on the nature of the deformation. We call this proportionality constant the **elastic modulus**. The elastic modulus is therefore defined as the ratio of the stress to the resulting strain:

$$\text{Elastic modulus} \equiv \frac{\text{stress}}{\text{strain}} \qquad (12.5)$$

The elastic modulus in general relates what is done to a solid object (a force is applied) to how that object responds (it deforms to some extent).

We consider three types of deformation and define an elastic modulus for each:

1. **Young's modulus**, which measures the resistance of a solid to a change in its length
2. **Shear modulus**, which measures the resistance to motion of the planes within a solid parallel to each other
3. **Bulk modulus**, which measures the resistance of solids or liquids to changes in their volume



Active Figure 12.14 A long bar clamped at one end is stretched by an amount ΔL under the action of a force **F**.

At the Active Figures link at http://www.pse6.com, you can adjust the values of the applied force and Young's modulus to observe the change in length of the bar.

Young's Modulus: Elasticity in Length

Consider a long bar of cross-sectional area A and initial length L_i that is clamped at one end, as in Figure 12.14. When an external force is applied perpendicular to the cross section, internal forces in the bar resist distortion ("stretching"), but the bar reaches an equilibrium situation in which its final length L_f is greater than L_i and in which the external force is exactly balanced by internal forces. In such a situation, the bar is said to be stressed. We define the **tensile stress** as the ratio of the magnitude of the external force F to the cross-sectional area A. The **tensile strain** in this case is defined as the ratio of the change in length ΔL to the original length L_i. We define **Young's modulus** by a combination of these two ratios:

Young's modulus

$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i} \qquad (12.6)$$

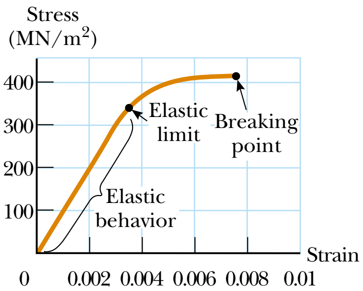
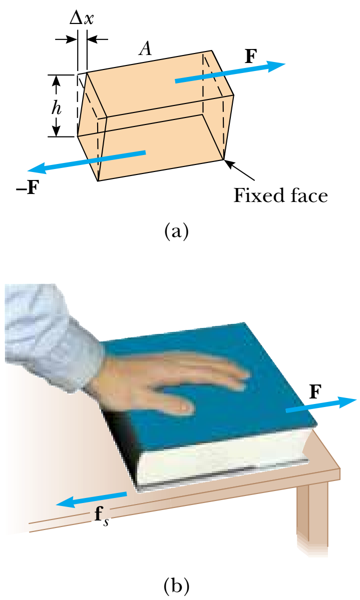


Figure 12.15 Stress-versus-strain curve for an elastic solid.



Active Figure 12.16 (a) A shear deformation in which a rectangular block is distorted by two forces of equal magnitude but opposite directions applied to two parallel faces. (b) A book under shear stress.

At the Active Figures link at http://www.pse6.com, you can adjust the values of the applied force and the shear modulus to observe the change in shape of the block in part (a).

Table 12.1

Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m²)	Shear Modulus (N/m²)	Bulk Modulus (N/m²)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5\text{–}7.8 \times 10^{10}$	$2.6\text{–}3.2 \times 10^{10}$	$5.0\text{–}5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression. Note that because strain is a dimensionless quantity, Y has units of force per unit area. Typical values are given in Table 12.1. Experiments show (a) that for a fixed applied force, the change in length is proportional to the original length and (b) that the force necessary to produce a given strain is proportional to the cross-sectional area. Both of these observations are in accord with Equation 12.6.

For relatively small stresses, the bar will return to its initial length when the force is removed. The **elastic limit** of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its initial length. It is possible to exceed the elastic limit of a substance by applying a sufficiently large stress, as seen in Figure 12.15. Initially, a stress-versus-strain curve is a straight line. As the stress increases, however, the curve is no longer a straight line. When the stress exceeds the elastic limit, the object is permanently distorted and does not return to its original shape after the stress is removed. As the stress is increased even further, the material ultimately breaks.

Shear Modulus: Elasticity of Shape

Another type of deformation occurs when an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force (Fig. 12.16a). The stress in this case is called a shear stress. If the object is originally a rectangular block, a shear stress results in a shape whose cross section is a parallelogram. A book pushed sideways, as shown in Figure 12.16b, is an example of an object subjected to a shear stress. To a first approximation (for small distortions), no change in volume occurs with this deformation.

We define the **shear stress** as F/A, the ratio of the tangential force to the area A of the face being sheared. The **shear strain** is defined as the ratio $\Delta x/h$, where Δx is the horizontal distance that the sheared face moves and h is the height of the object. In terms of these quantities, the **shear modulus** is

Shear modulus

$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h} \tag{12.7}$$

Values of the shear modulus for some representative materials are given in Table 12.1. Like Young's modulus, the unit of shear modulus is the ratio of that for force to that for area.

Bulk Modulus: Volume Elasticity

Bulk modulus characterizes the response of an object to changes in a force of uniform magnitude applied perpendicularly over the entire surface of the object, as shown in Figure 12.17. (We assume here that the object is made of a single substance.) As we shall see in Chapter 14, such a uniform distribution of forces occurs when an object is immersed in a fluid. An object subject to this type of deformation undergoes a change in volume but no change in shape. The **volume stress** is defined as the ratio of the magnitude of the total force F exerted on a surface to the area A of the surface. The quantity $P = F/A$ is called **pressure**, which we will study in more detail in Chapter 14. If the pressure on an object changes by an amount $\Delta P = \Delta F/A$, then the object will experience a volume change ΔV. The **volume strain** is equal to the change in volume ΔV divided by the initial volume V_i. Thus, from Equation 12.5, we can characterize a volume ("bulk") compression in terms of the **bulk modulus**, which is defined as

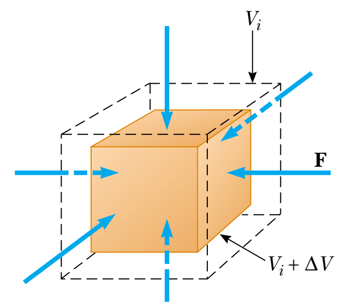
$$B \equiv \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i} \qquad (12.8)$$

Bulk modulus

A negative sign is inserted in this defining equation so that B is a positive number. This maneuver is necessary because an increase in pressure (positive ΔP) causes a decrease in volume (negative ΔV) and vice versa.

Table 12.1 lists bulk moduli for some materials. If you look up such values in a different source, you often find that the reciprocal of the bulk modulus is listed. The reciprocal of the bulk modulus is called the **compressibility** of the material.

Note from Table 12.1 that both solids and liquids have a bulk modulus. However, no shear modulus and no Young's modulus are given for liquids because a liquid does not sustain a shearing stress or a tensile stress. If a shearing force or a tensile force is applied to a liquid, the liquid simply flows in response.



Active Figure 12.17 When a solid is under uniform pressure, it undergoes a change in volume but no change in shape. This cube is compressed on all sides by forces normal to its six faces.

At the Active Figures link at http://www.pse6.com, you can adjust the values of the applied force and the bulk modulus to observe the change in volume of the cube.

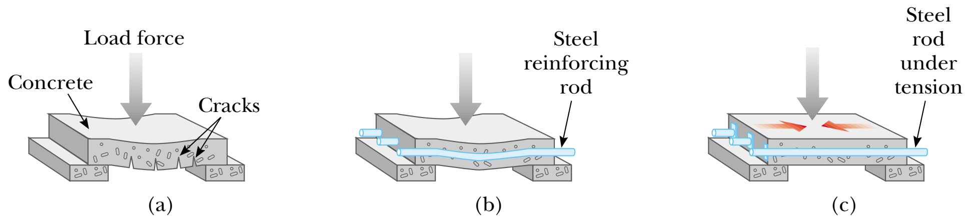
Quick Quiz 12.4 A block of iron is sliding across a horizontal floor. The friction force between the block and the floor causes the block to deform. To describe the relationship between stress and strain for the block, you would use (a) Young's modulus (b) shear modulus (c) bulk modulus (d) none of these.

Quick Quiz 12.5 A trapeze artist swings through a circular arc. At the bottom of the swing, the wires supporting the trapeze are longer than when the trapeze artist simply hangs from the trapeze, due to the increased tension in them. To describe the relationship between stress and strain for the wires, you would use (a) Young's modulus (b) shear modulus (c) bulk modulus (d) none of these.

Quick Quiz 12.6 A spacecraft carries a steel sphere to a planet on which atmospheric pressure is much higher than on the Earth. The higher pressure causes the radius of the sphere to decrease. To describe the relationship between stress and strain for the sphere, you would use (a) Young's modulus (b) shear modulus (c) bulk modulus (d) none of these.

Prestressed Concrete

If the stress on a solid object exceeds a certain value, the object fractures. The maximum stress that can be applied before fracture occurs depends on the nature of the material and on the type of applied stress. For example, concrete has a tensile strength of about 2×10^6 N/m^2, a compressive strength of 20×10^6 N/m^2, and a shear strength of 2×10^6 N/m^2. If the applied stress exceeds these values, the concrete fractures. It is common practice to use large safety factors to prevent failure in concrete structures.



Active Figure 12.18 (a) A concrete slab with no reinforcement tends to crack under a heavy load. (b) The strength of the concrete is increased by using steel reinforcement rods. (c) The concrete is further strengthened by prestressing it with steel rods under tension.

Concrete is normally very brittle when it is cast in thin sections. Thus, concrete slabs tend to sag and crack at unsupported areas, as shown in Figure 12.18a. The slab can be strengthened by the use of steel rods to reinforce the concrete, as illustrated in Figure 12.18b. Because concrete is much stronger under compression (squeezing) than under tension (stretching) or shear, vertical columns of concrete can support very heavy loads, whereas horizontal beams of concrete tend to sag and crack. However, a significant increase in shear strength is achieved if the reinforced concrete is prestressed, as shown in Figure 12.18c. As the concrete is being poured, the steel rods are held under tension by external forces. The external forces are released after the concrete cures; this results in a permanent tension in the steel and hence a compressive stress on the concrete. This enables the concrete slab to support a much heavier load.

Example 12.6 Stage Design

Recall Example 8.4, in which we analyzed a cable used to support an actor as he swung onto the stage. Suppose that the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10-m-long steel wire have if we do not want it to stretch more than 0.5 cm under these conditions?

Solution From the definition of Young's modulus, we can solve for the required cross-sectional area. Assuming that the cross section is circular, we can determine the diameter of the wire. From Equation 12.6, we have

$$Y = \frac{F/A}{\Delta L/L_i}$$

$$A = \frac{FL_i}{Y\Delta L} = \frac{(940\text{ N})(10\text{ m})}{(20 \times 10^{10}\text{ N/m}^2)(0.005\text{ m})}$$

$$= 9.4 \times 10^{-6}\text{ m}^2$$

Because $A = \pi r^2$, the radius of the wire can be found from

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{9.4 \times 10^{-6}\text{ m}^2}{\pi}} = 1.7 \times 10^{-3}\text{ m} = 1.7\text{ mm}$$

$$d = 2r = 2(1.7\text{ mm}) = \boxed{3.4\text{ mm}}$$

To provide a large margin of safety, we would probably use a flexible cable made up of many smaller wires having a total cross-sectional area substantially greater than our calculated value.

Example 12.7 Squeezing a Brass Sphere

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is $1.0 \times 10^5\text{ N/m}^2$ (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is $2.0 \times 10^7\text{ N/m}^2$. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

Solution From the definition of bulk modulus, we have

$$B = -\frac{\Delta P}{\Delta V/V_i}$$

$$\Delta V = -\frac{V_i \Delta P}{B}$$

Substituting the numerical values, we obtain

$$\Delta V = -\frac{(0.50\text{ m}^3)(2.0 \times 10^7\text{ N/m}^2 - 1.0 \times 10^5\text{ N/m}^2)}{6.1 \times 10^{10}\text{ N/m}^2}$$

$$= \boxed{-1.6 \times 10^{-4}\text{ m}^3}$$

The negative sign indicates that the volume of the sphere decreases.

SUMMARY

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

A rigid object is in **equilibrium** if and only if **the resultant external force acting on it is zero and the resultant external torque on it is zero about any axis:**

$$\sum \mathbf{F} = 0 \tag{12.1}$$

$$\sum \boldsymbol{\tau} = 0 \tag{12.2}$$

The first condition is the condition for translational equilibrium, and the second is the condition for rotational equilibrium. These two equations allow you to analyze a great variety of problems. Make sure you can identify forces unambiguously, create a free-body diagram, and then apply Equations 12.1 and 12.2 and solve for the unknowns.

The gravitational force exerted on an object can be considered as acting at a single point called the **center of gravity**. The center of gravity of an object coincides with its center of mass if the object is in a uniform gravitational field.

We can describe the elastic properties of a substance using the concepts of stress and strain. **Stress** is a quantity proportional to the force producing a deformation; **strain** is a measure of the degree of deformation. Strain is proportional to stress, and the constant of proportionality is the **elastic modulus:**

$$\text{Elastic modulus} \equiv \frac{\text{stress}}{\text{strain}} \tag{12.5}$$

Three common types of deformation are represented by (1) the resistance of a solid to elongation under a load, characterized by **Young's modulus** Y; (2) the resistance of a solid to the motion of internal planes sliding past each other, characterized by the **shear modulus** S; and (3) the resistance of a solid or fluid to a volume change, characterized by the **bulk modulus** B.

QUESTIONS

1. Stand with your back against a wall. Why can't you put your heels firmly against the wall and then bend forward without falling?
2. Can an object be in equilibrium if it is in motion? Explain.
3. Can an object be in equilibrium when only one force acts upon it? If you believe the answer is yes, give an example to support your conclusion.
4. (a) Give an example in which the net force acting on an object is zero and yet the net torque is nonzero. (b) Give an example in which the net torque acting on an object is zero and yet the net force is nonzero.
5. Can an object be in equilibrium if the only torques acting on it produce clockwise rotation?
6. If you measure the net force and the net torque on a system to be zero, (a) could the system still be rotating with respect to you? (b) Could it be translating with respect to you?
7. The center of gravity of an object may be located outside the object. Give a few examples for which this is the case.
8. Assume you are given an arbitrarily shaped piece of plywood, together with a hammer, nail, and plumb bob. How could you use these items to locate the center of gravity of the plywood? *Suggestion:* Use the nail to suspend the plywood.
9. For a chair to be balanced on one leg, where must the center of gravity of the chair be located?
10. A girl has a large, docile dog she wishes to weigh on a small bathroom scale. She reasons that she can determine her dog's weight with the following method: First she puts the dog's two front feet on the scale and records the scale reading. Then she places the dog's two back feet on the scale and records the reading. She thinks that the sum of the readings will be the dog's weight. Is she correct? Explain your answer.
11. A tall crate and a short crate of equal mass are placed side by side on an incline, without touching each other. As the incline angle is increased, which crate will topple first? Explain.
12. A ladder stands on the ground, leaning against a wall. Would you feel safer climbing up the ladder if you were told that the ground is frictionless but the wall is rough, or that the wall is frictionless but the ground is rough? Justify your answer.
13. When you are lifting a heavy object, it is recommended that you keep your back as nearly vertical as possible, lifting from your knees. Why is this better than bending over and lifting from your waist?
14. What kind of deformation does a cube of Jell-O exhibit when it jiggles?
15. Ruins of ancient Greek temples often have intact vertical columns, but few horizontal slabs of stone are still in place. Can you think of a reason why this is so?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*
= coached solution with hints available at http://www.pse6.com = computer useful in solving problem
= paired numerical and symbolic problems

Section 12.1 The Conditions for Equilibrium of a Rigid Body

1. A baseball player holds a 36-oz bat (weight = 10.0 N) with one hand at the point O (Fig. P12.1). The bat is in equilibrium. The weight of the bat acts along a line 60.0 cm to the right of O. Determine the force and the torque exerted by the player on the bat around an axis through O.

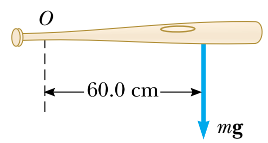


Figure P12.1

2. Write the necessary conditions for equilibrium of the object shown in Figure P12.2. Take the origin of the torque equation at the point O.

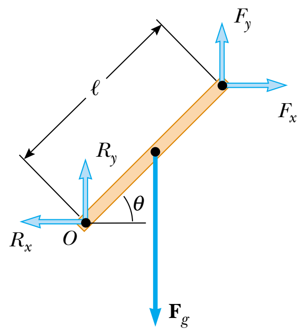


Figure P12.2

3. A uniform beam of mass m_b and length ℓ supports blocks with masses m_1 and m_2 at two positions, as in Figure P12.3. The beam rests on two knife edges. For what value of x will the beam be balanced at P such that the normal force at O is zero?

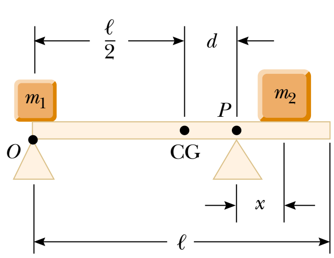


Figure P12.3

Section 12.2 More on the Center of Gravity

Problems 38, 39, 41, 43, and 44 in Chapter 9 can also be assigned with this section.

4. A circular pizza of radius R has a circular piece of radius $R/2$ removed from one side as shown in Figure P12.4. The center of gravity has moved from C to C' along the x axis. Show that the distance from C to C' is $R/6$. Assume the thickness and density of the pizza are uniform throughout.

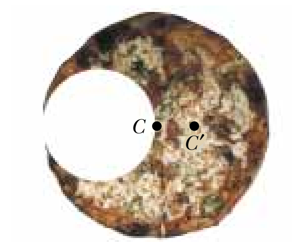


Figure P12.4

5. A carpenter's square has the shape of an L, as in Figure P12.5. Locate its center of gravity.

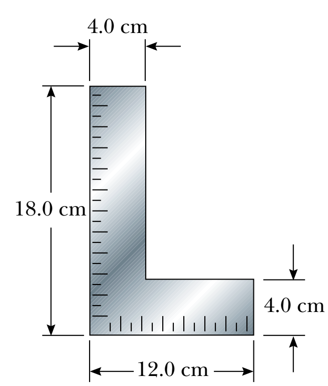


Figure P12.5

6. Pat builds a track for his model car out of wood, as in Figure P12.6. The track is 5.00 cm wide, 1.00 m high, and 3.00 m

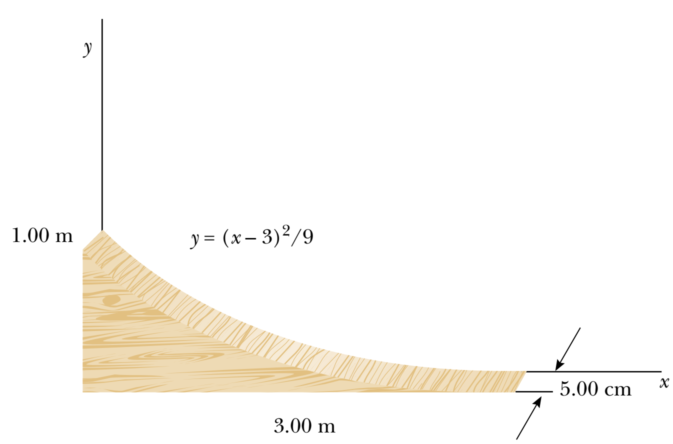


Figure P12.6

long, and is solid. The runway is cut such that it forms a parabola with the equation $y = (x - 3)^2/9$. Locate the horizontal coordinate of the center of gravity of this track.

7. Consider the following mass distribution: 5.00 kg at (0, 0) m, 3.00 kg at (0, 4.00) m, and 4.00 kg at (3.00, 0) m. Where should a fourth object of mass 8.00 kg be placed so that the center of gravity of the four-object arrangement will be at (0, 0)?

8. Figure P12.8 shows three uniform objects: a rod, a right triangle, and a square. Their masses and their coordinates in meters are given. Determine the center of gravity for the three-object system.

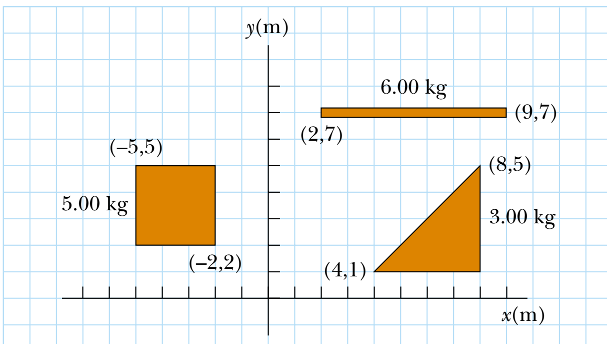


Figure P12.8

Section 12.3 Examples of Rigid Objects in Static Equilibrium

Problems 17, 18, 19, 20, 21, 27, 40, 46, 57, 59, and 73 in Chapter 5 can also be assigned with this section.

9. Find the mass m of the counterweight needed to balance the 1 500-kg truck on the incline shown in Figure P12.9. Assume all pulleys are frictionless and massless.

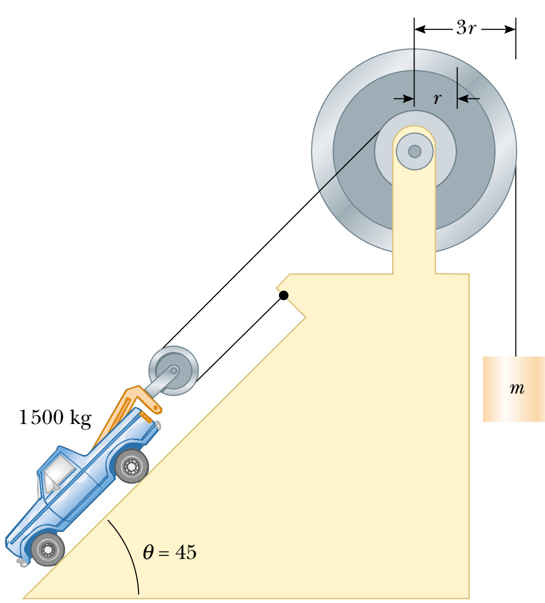


Figure P12.9

10. A mobile is constructed of light rods, light strings, and beach souvenirs, as shown in Figure P12.10. Determine the masses of the objects (a) m_1, (b) m_2, and (c) m_3.

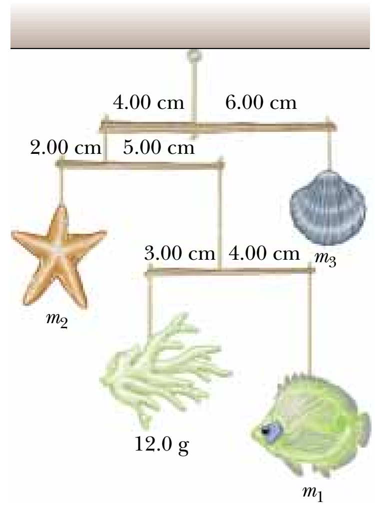


Figure P12.10

11. Two pans of a balance are 50.0 cm apart. The fulcrum of the balance has been shifted 1.00 cm away from the center by a dishonest shopkeeper. By what percentage is the true weight of the goods being marked up by the shopkeeper? (Assume the balance has negligible mass.)

12. A 20.0-kg floodlight in a park is supported at the end of a horizontal beam of negligible mass that is hinged to a pole, as shown in Figure P12.12. A cable at an angle of 30.0° with the beam helps to support the light. Find (a) the tension in the cable and (b) the horizontal and vertical forces exerted on the beam by the pole.

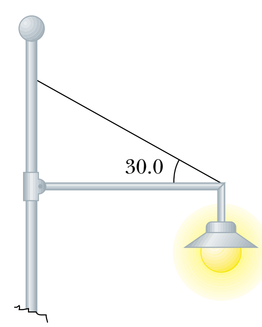


Figure P12.12

13. A 15.0-m uniform ladder weighing 500 N rests against a frictionless wall. The ladder makes a 60.0° angle with the horizontal. (a) Find the horizontal and vertical forces the ground exerts on the base of the ladder when an 800-N firefighter is 4.00 m from the bottom. (b) If the ladder is just on the verge of slipping when the firefighter is 9.00 m up, what is the coefficient of static friction between ladder and ground?

14. A uniform ladder of length L and mass m_1 rests against a frictionless wall. The ladder makes an angle θ with the horizontal. (a) Find the horizontal and vertical forces the ground exerts on the base of the ladder when a firefighter of mass m_2 is a distance x from the bottom. (b) If the ladder is just on the verge of slipping when the firefighter is a distance d from the bottom, what is the coefficient of static friction between ladder and ground?

15. Figure P12.15 shows a claw hammer as it is being used to pull a nail out of a horizontal board. If a force of 150 N is exerted horizontally as shown, find (a) the force exerted by the hammer claws on the nail and (b) the force exerted by the surface on the point of contact with the hammer head. Assume that the force the hammer exerts on the nail is parallel to the nail.

Figure P12.15

16. A uniform plank of length 6.00 m and mass 30.0 kg rests horizontally across two horizontal bars of a scaffold. The bars are 4.50 m apart, and 1.50 m of the plank hangs over one side of the scaffold. Draw a free-body diagram of the plank. How far can a painter of mass 70.0 kg walk on the overhanging part of the plank before it tips?

$\boxed{17.}$ A 1 500-kg automobile has a wheel base (the distance between the axles) of 3.00 m. The center of mass of the automobile is on the center line at a point 1.20 m behind the front axle. Find the force exerted by the ground on each wheel.

18. A vertical post with a square cross section is 10.0 m tall. Its bottom end is encased in a base 1.50 m tall, which is precisely square but slightly loose. A force 5.50 N to the right acts on the top of the post. The base maintains the post in equilibrium. Find the force that the top of the right side wall of the base exerts on the post. Find the force that the bottom of the left side wall of the base exerts on the post.

19. A flexible chain weighing 40.0 N hangs between two hooks located at the same height (Fig. P12.19). At each hook, the tangent to the chain makes an angle $\theta = 42.0°$ with the horizontal. Find (a) the magnitude of the force each hook exerts on the chain and (b) the tension in the chain at its midpoint. (*Suggestion:* for part (b), make a free-body diagram for half of the chain.)

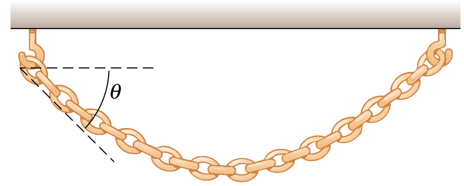


Figure P12.19

20. Sir Lost-a-Lot dons his armor and sets out from the castle on his trusty steed in his quest to improve communication between damsels and dragons (Fig. P12.20). Unfortunately his squire lowered the drawbridge too far and finally stopped it 20.0° below the horizontal. Lost-a-Lot and his horse stop when their combined center of mass is 1.00 m from the end of the bridge. The uniform bridge is 8.00 m long and has mass 2 000 kg. The lift cable is attached to the bridge 5.00 m from the hinge at the castle end, and to a point on the castle wall 12.0 m above the bridge. Lost-a-Lot's mass combined with his armor and steed is 1 000 kg. Determine (a) the tension in the cable and the (b) horizontal and (c) vertical force components acting on the bridge at the hinge.

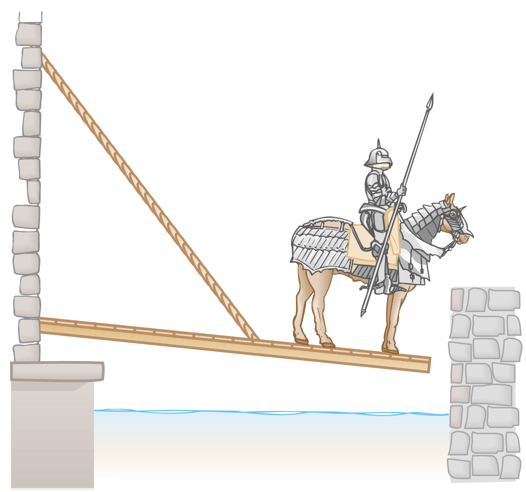


Figure P12.20 Problems 20 and 21.

21. Review problem. In the situation described in Problem 20 and illustrated in Figure P12.20, the lift cable suddenly breaks! The hinge between the castle wall and the bridge is frictionless, and the bridge swings freely until it is vertical. (a) Find the angular acceleration of the bridge once it starts to move. (b) Find the angular speed of the bridge when it strikes the vertical castle wall below the hinge. (c) Find the force exerted by the hinge on the bridge immediately after the cable breaks. (d) Find the force exerted by the hinge on the bridge immediately before it strikes the castle wall.

22. Stephen is pushing his sister Joyce in a wheelbarrow when it is stopped by a brick 8.00 cm high (Fig. P12.22).

The handles make an angle of 15.0° below the horizontal. A downward force of 400 N is exerted on the wheel, which has a radius of 20.0 cm. (a) What force must Stephen apply along the handles in order to just start the wheel over the brick? (b) What is the force (magnitude and direction) that the brick exerts on the wheel just as the wheel begins to lift over the brick? Assume in both parts that the brick remains fixed and does not slide along the ground.

Figure P12.22

23. One end of a uniform 4.00-m-long rod of weight F_g is supported by a cable. The other end rests against the wall, where it is held by friction, as in Figure P12.23. The coefficient of static friction between the wall and the rod is $\mu_s = 0.500$. Determine the minimum distance x from point A at which an additional weight F_g (the same as the weight of the rod) can be hung without causing the rod to slip at point A.

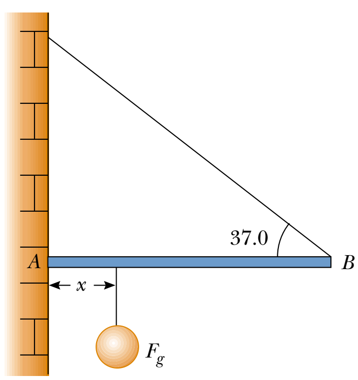


Figure P12.23

24. Two identical uniform bricks of length L are placed in a stack over the edge of a horizontal surface with the maximum overhang possible without falling, as in Figure P12.24. Find the distance x.

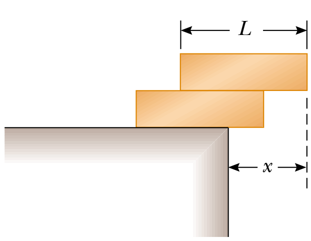


Figure P12.24

25. A vaulter holds a 29.4-N pole in equilibrium by exerting an upward force **U** with her leading hand and a downward force **D** with her trailing hand, as shown in Figure P12.25. Point C is the center of gravity of the pole. What are the magnitudes of **U** and **D**?

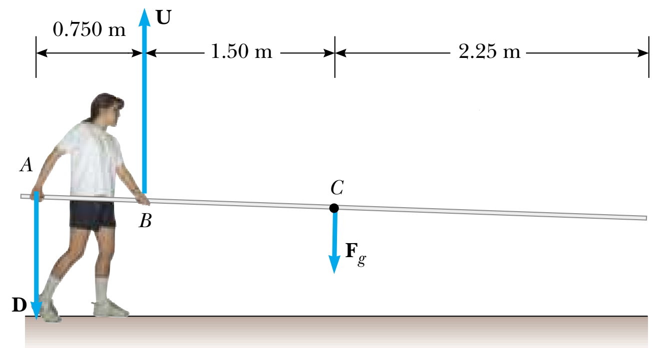


Figure P12.25

26. In the **What If?** section of Example 12.3, let x represent the distance in meters between the person and the hinge at the left end of the beam. (a) Show that the cable tension in newtons is given by $T = 93.9x + 125$. Argue that T increases as x increases. (b) Show that the direction angle θ of the hinge force is described by

$$\tan\,\theta = \left(\frac{32}{3x + 4} - 1\right)\tan\,53.0°$$

How does θ change as x increases? (c) Show that the magnitude of the hinge force is given by

$$R = \sqrt{8.82 \times 10^3 x^2 - 9.65 \times 10^4 x + 4.96 \times 10^5}$$

How does R change as x increases?

Section 12.4 Elastic Properties of Solids

27. A 200-kg load is hung on a wire having a length of 4.00 m, cross-sectional area 0.200×10^{-4} m^2, and Young's modulus 8.00×10^{10} N/m^2. What is its increase in length?

28. Assume that Young's modulus is 1.50×10^{10} N/m^2 for bone and that the bone will fracture if stress greater than 1.50×10^8 N/m^2 is imposed on it. (a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm? (b) If this much force is applied compressively, by how much does the 25.0-cm-long bone shorten?

29. Evaluate Young's modulus for the material whose stress-versus-strain curve is shown in Figure 12.15.

30. A steel wire of diameter 1 mm can support a tension of 0.2 kN. A cable to support a tension of 20 kN should have diameter of what order of magnitude?

31. A child slides across a floor in a pair of rubber-soled shoes. The friction force acting on each foot is 20.0 N.

The footprint area of each shoe sole is 14.0 cm^2, and the thickness of each sole is 5.00 mm. Find the horizontal distance by which the upper and lower surfaces of each sole are offset. The shear modulus of the rubber is 3.00 MN/m^2.

32. **Review problem.** A 30.0-kg hammer strikes a steel spike 2.30 cm in diameter while moving with speed 20.0 m/s. The hammer rebounds with speed 10.0 m/s after 0.110 s. What is the average strain in the spike during the impact?

33. If the shear stress in steel exceeds 4.00×10^8 N/m^2, the steel ruptures. Determine the shearing force necessary to (a) shear a steel bolt 1.00 cm in diameter and (b) punch a 1.00-cm-diameter hole in a steel plate 0.500 cm thick.

34. **Review problem.** A 2.00-m-long cylindrical steel wire with a cross-sectional diameter of 4.00 mm is placed over a light frictionless pulley, with one end of the wire connected to a 5.00-kg object and the other end connected to a 3.00-kg object. By how much does the wire stretch while the objects are in motion?

35. When water freezes, it expands by about 9.00%. What pressure increase would occur inside your automobile engine block if the water in it froze? (The bulk modulus of ice is 2.00×10^9 N/m^2.)

36. The deepest point in the ocean is in the Mariana Trench, about 11 km deep. The pressure at this depth is huge, about 1.13×10^8 N/m^2. (a) Calculate the change in volume of 1.00 m^3 of seawater carried from the surface to this deepest point in the Pacific ocean. (b) The density of seawater at the surface is 1.03×10^3 kg/m^3. Find its density at the bottom. (c) Is it a good approximation to think of water as incompressible?

37. A walkway suspended across a hotel lobby is supported at numerous points along its edges by a vertical cable above each point and a vertical column underneath. The steel cable is 1.27 cm in diameter and is 5.75 m long before loading. The aluminum column is a hollow cylinder with an inside diameter of 16.14 cm, an outside diameter of 16.24 cm, and unloaded length of 3.25 m. When the walkway exerts a load force of 8 500 N on one of the support points, how much does the point move down?

Additional Problems

38. A lightweight, rigid beam 10.0 m long is supported by a cable attached to a spring of force constant $k = 8.25$ kN/m as shown in Figure P12.38. When no load is hung on the beam ($F_g = 0$), the length L is equal to 5.00 m. (a) Find the angle θ in this situation. (b) Now a load of $F_g = 250$ N is hung on the end of the beam. Temporarily ignore the extension of the spring and the change in the angle θ. Calculate the tension in the cable with this approximation. (c) Use the answer to part (b) to calculate the spring elongation and a new value for the angle θ. (d) With the value of θ from part (c), find a second approximation for the tension in the cable. (e) Use the answer to part (d) to calculate more precise values for the spring elongation and the angle θ. (f) To three-digit precision, what is the actual value of θ under load?

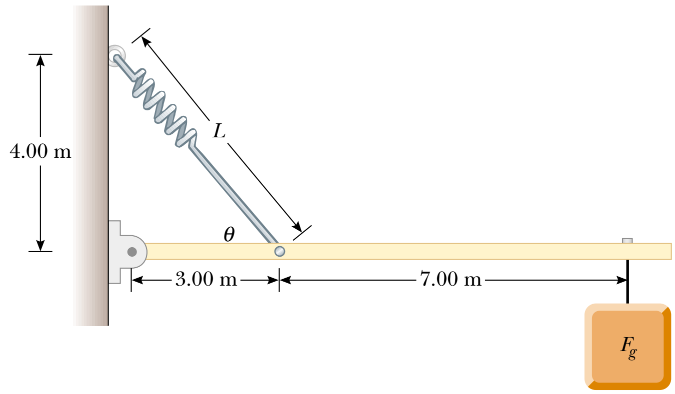


Figure P12.38

39. A bridge of length 50.0 m and mass 8.00×10^4 kg is supported on a smooth pier at each end as in Figure P12.39. A truck of mass 3.00×10^4 kg is located 15.0 m from one end. What are the forces on the bridge at the points of support?

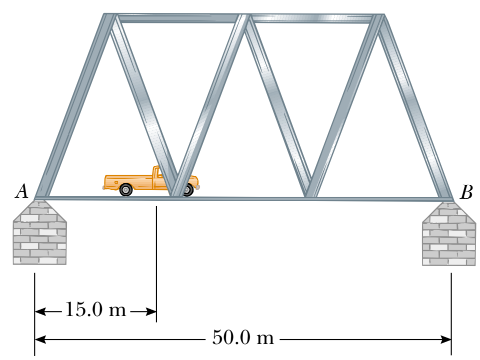


Figure P12.39

40. Refer to Figure 12.18(c). A lintel of prestressed reinforced concrete is 1.50 m long. The cross-sectional area of the concrete is 50.0 cm^2. The concrete encloses one steel reinforcing rod with cross-sectional area 1.50 cm^2. The rod joins two strong end plates. Young's modulus for the concrete is 30.0×10^9 N/m^2. After the concrete cures and the original tension T_1 in the rod is released, the concrete is to be under compressive stress 8.00×10^6 N/m^2. (a) By what distance will the rod compress the concrete when the original tension in the rod is released? (b) The rod will still be under what tension T_2? (c) The rod will then be how much longer than its unstressed length? (d) When the concrete was poured, the rod should have been stretched by what extension distance from its unstressed length? (e) Find the required original tension T_1 in the rod.

41. A uniform pole is propped between the floor and the ceiling of a room. The height of the room is 7.80 ft, and the coefficient of static friction between the pole and the ceiling is 0.576. The coefficient of static friction between the pole and the floor is greater than that. What is the length of the longest pole that can be propped between the floor and the ceiling?

42. A solid sphere of radius R and mass M is placed in a trough as shown in Figure P12.42. The inner surfaces of the trough are frictionless. Determine the forces exerted by the trough on the sphere at the two contact points.

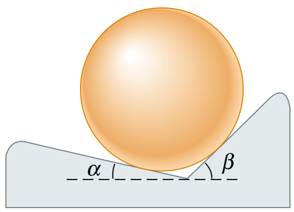


Figure P12.42

43. A hungry bear weighing 700 N walks out on a beam in an attempt to retrieve a basket of food hanging at the end of the beam (Fig. P12.43). The beam is uniform, weighs 200 N, and is 6.00 m long; the basket weighs 80.0 N. (a) Draw a free-body diagram for the beam. (b) When the bear is at $x = 1.00$ m, find the tension in the wire and the components of the force exerted by the wall on the left end of the beam. (c) **What If?** If the wire can withstand a maximum tension of 900 N, what is the maximum distance the bear can walk before the wire breaks?

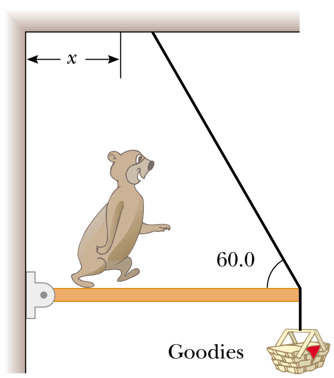


Figure P12.43

44. A farm gate (Fig. P12.44) is 3.00 m wide and 1.80 m high, with hinges attached to the top and bottom. The guy wire makes an angle of 30.0° with the top of the gate and is tightened by a turnbuckle to a tension of 200 N. The mass of the gate is 40.0 kg. (a) Determine the horizontal force exerted by the bottom hinge on the gate. (b) Find the horizontal force exerted by the upper hinge. (c) Determine the combined vertical force exerted by both hinges. (d) **What If?** What must be the tension in the guy wire so that the horizontal force exerted by the upper hinge is zero?

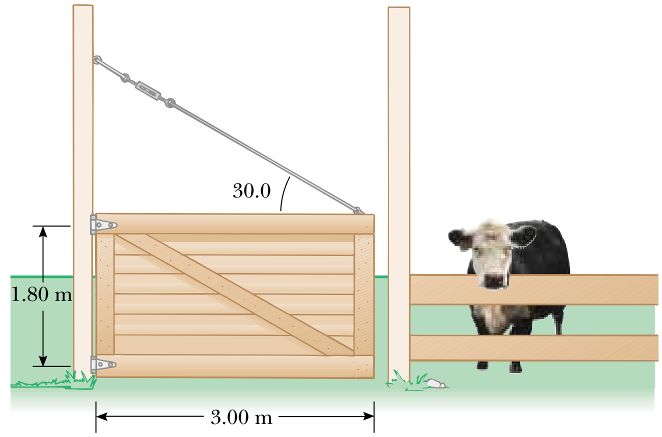


Figure P12.44

45. A uniform sign of weight F_g and width $2L$ hangs from a light, horizontal beam, hinged at the wall and supported by a cable (Fig. P12.45). Determine (a) the tension in the cable and (b) the components of the reaction force exerted by the wall on the beam, in terms of F_g, d, L, and θ.

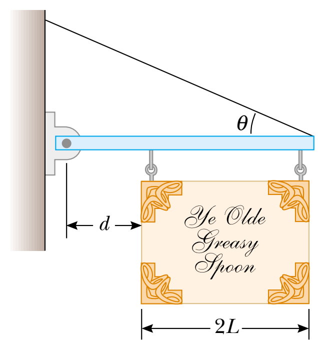


Figure P12.45

46. A 1 200-N uniform boom is supported by a cable as in Figure P12.46. The boom is pivoted at the bottom, and a 2 000-N object hangs from its top. Find the tension in the cable and the components of the reaction force exerted by the floor on the boom.

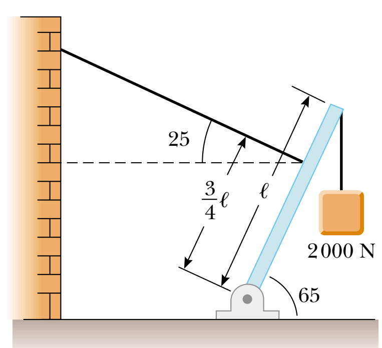


Figure P12.46

47. A crane of mass 3 000 kg supports a load of 10 000 kg as in Figure P12.47. The crane is pivoted with a frictionless pin at A and rests against a smooth support at B. Find the reaction forces at A and B.

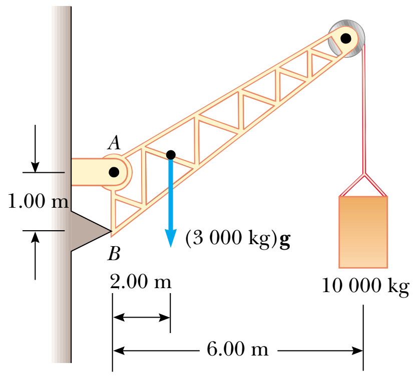


Figure P12.47

48. A ladder of uniform density and mass m rests against a frictionless vertical wall, making an angle of 60.0° with the horizontal. The lower end rests on a flat surface where the coefficient of static friction is $\mu_s = 0.400$. A window cleaner with mass $M = 2m$ attempts to climb the ladder. What fraction of the length L of the ladder will the worker have reached when the ladder begins to slip?

49. A 10 000-N shark is supported by a cable attached to a 4.00-m rod that can pivot at the base. Calculate the tension in the tie-rope between the rod and the wall if it is holding the system in the position shown in Figure P12.49. Find the horizontal and vertical forces exerted on the base of the rod. (Neglect the weight of the rod.)

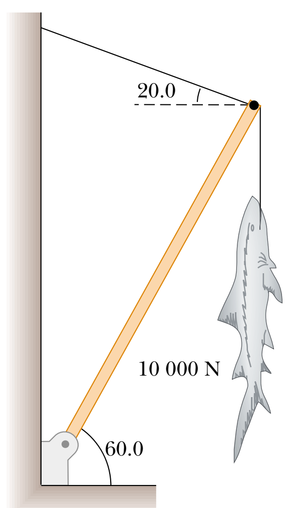


Figure P12.49

50. When a person stands on tiptoe (a strenuous position), the position of the foot is as shown in Figure P12.50a. The gravitational force on the body $\mathbf{F}_g$ is supported by the force $\mathbf{n}$ exerted by the floor on the toe. A mechanical model for the situation is shown in Figure P12.50b, where $\mathbf{T}$ is the force exerted by the Achilles tendon on the foot and $\mathbf{R}$ is the force exerted by the tibia on the foot. Find the values of T, R, and θ when $F_g = 700$ N.

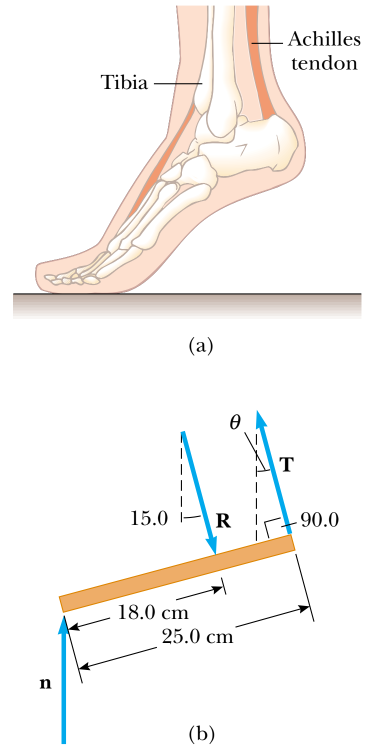


Figure P12.50

51. A person bending forward to lift a load "with his back" (Fig. P12.51a) rather than "with his knees" can be injured by large forces exerted on the muscles and vertebrae. The spine pivots mainly at the fifth lumbar vertebra, with the principal supporting force provided by the erector spinalis muscle in the back. To see the magnitude of the forces involved, and to understand why back problems are common among humans, consider the model shown in Figure P12.51b for a person bending forward to lift a 200-N object. The spine and upper body are represented as a uniform horizontal rod of weight 350 N, pivoted at the base of the spine. The erector spinalis muscle, attached at a

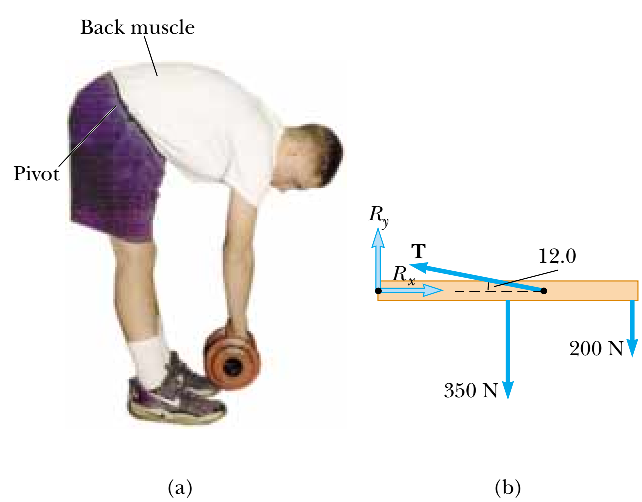


Figure P12.51

point two thirds of the way up the spine, maintains the position of the back. The angle between the spine and this muscle is 12.0°. Find the tension in the back muscle and the compressional force in the spine.

52. A uniform rod of weight F_g and length L is supported at its ends by a frictionless trough as shown in Figure P12.52. (a) Show that the center of gravity of the rod must be vertically over point O when the rod is in equilibrium. (b) Determine the equilibrium value of the angle θ.

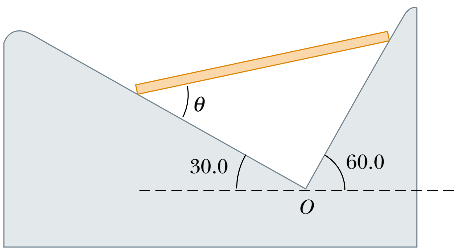


Figure P12.52

53. A force acts on a rectangular cabinet weighing 400 N, as in Figure P12.53. (a) If the cabinet slides with constant speed when $F = 200$ N and $h = 0.400$ m, find the coefficient of kinetic friction and the position of the resultant normal force. (b) If $F = 300$ N, find the value of h for which the cabinet just begins to tip.

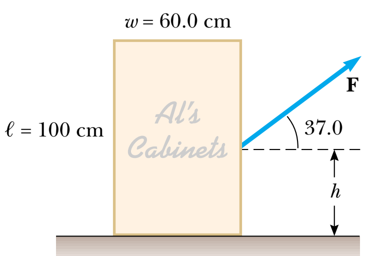


Figure P12.53 Problems 53 and 54.

54. Consider the rectangular cabinet of Problem 53, but with a force **F** applied horizontally at the upper edge. (a) What is the minimum force required to start to tip the cabinet? (b) What is the minimum coefficient of static friction required for the cabinet not to slide with the application of a force of this magnitude? (c) Find the magnitude and direction of the minimum force required to tip the cabinet if the point of application can be chosen anywhere on the cabinet.

55. A uniform beam of mass m is inclined at an angle θ to the horizontal. Its upper end produces a ninety-degree bend in a very rough rope tied to a wall, and its lower end rests on a rough floor (Fig. P12.55). (a) If the coefficient of static friction between beam and floor is μ_s, determine an expression for the maximum mass M that can be suspended from the top before the beam slips. (b) Determine the magnitude of the reaction force at the floor and the magnitude of the force exerted by the beam on the rope at P in terms of m, M, and μ_s.

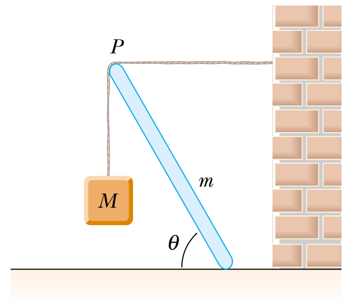


Figure P12.55

56. Figure P12.56 shows a truss that supports a downward force of 1 000 N applied at the point B. The truss has negligible weight. The piers at A and C are smooth. (a) Apply the conditions of equilibrium to prove that $n_A = 366$ N and $n_C = 634$ N. (b) Show that, because forces act on the light truss only at the hinge joints, each bar of the truss must exert on each hinge pin only a force along the length of that bar—a force of tension or compression. (c) Find the force of tension or of compression in each of the three bars.

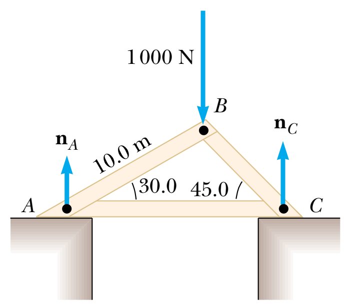


Figure P12.56

57. A stepladder of negligible weight is constructed as shown in Figure P12.57. A painter of mass 70.0 kg stands on the ladder 3.00 m from the bottom. Assuming the floor is frictionless, find (a) the tension in the horizontal bar connecting the two halves of the ladder, (b) the normal forces at A

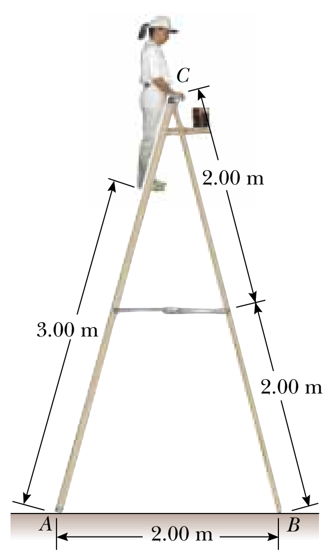


Figure P12.57

and B, and (c) the components of the reaction force at the single hinge C that the left half of the ladder exerts on the right half. (*Suggestion:* Treat the ladder as a single object, but also each half of the ladder separately.)

58. A flat dance floor of dimensions 20.0 m by 20.0 m has a mass of 1 000 kg. Three dance couples, each of mass 125 kg, start in the top left, top right, and bottom left corners. (a) Where is the initial center of gravity? (b) The couple in the bottom left corner moves 10.0 m to the right. Where is the new center of gravity? (c) What was the average velocity of the center of gravity if it took that couple 8.00 s to change positions?

59. A shelf bracket is mounted on a vertical wall by a single screw, as shown in Figure P12.59. Neglecting the weight of the bracket, find the horizontal component of the force that the screw exerts on the bracket when an 80.0 N vertical force is applied as shown. (*Hint:* Imagine that the bracket is slightly loose.)

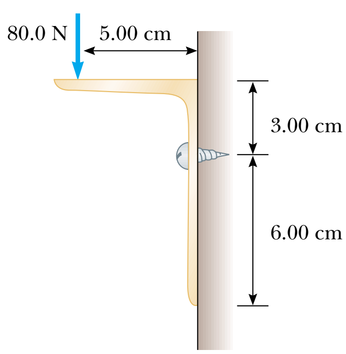


Figure P12.59

60. Figure P12.60 shows a vertical force applied tangentially to a uniform cylinder of weight F_g. The coefficient of static friction between the cylinder and all surfaces is 0.500. In terms of F_g, find the maximum force **P** that can be applied that does not cause the cylinder to rotate. (*Hint:* When the cylinder is on the verge of slipping, both friction forces are at their maximum values. Why?)

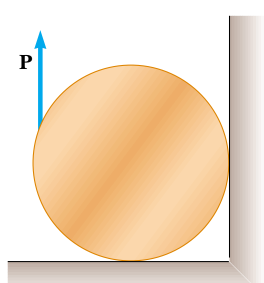


Figure P12.60

61. **Review problem.** A wire of length L, Young's modulus Y, and cross-sectional area A is stretched elastically by an amount ΔL. By Hooke's law (Section 7.4), the restoring force is $-k\,\Delta L$. (a) Show that $k = YA/L$. (b) Show that the work done in stretching the wire by an amount ΔL is

$$W = \tfrac{1}{2}YA(\Delta L)^2/L$$

62. Two racquetballs are placed in a glass jar, as shown in Figure P12.62. Their centers and the point A lie on a straight line. (a) Assume that the walls are frictionless, and determine P_1, P_2, and P_3. (b) Determine the magnitude of the force exerted by the left ball on the right ball. Assume each ball has a mass of 170 g.

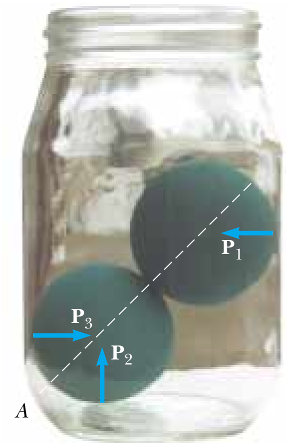


Figure P12.62

63. In exercise physiology studies it is sometimes important to determine the location of a person's center of mass. This can be done with the arrangement shown in Figure P12.63. A light plank rests on two scales, which give readings of $F_{g1} = 380$ N and $F_{g2} = 320$ N. The scales are separated by a distance of 2.00 m. How far from the woman's feet is her center of mass?

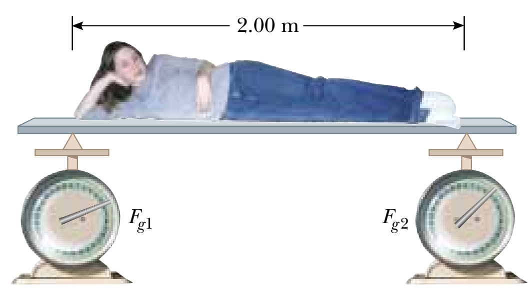


Figure P12.63

64. A steel cable 3.00 cm^2 in cross-sectional area has a mass of 2.40 kg per meter of length. If 500 m of the cable is hung over a vertical cliff, how much does the cable stretch under its own weight? $Y_{steel} = 2.00 \times 10^{11}$ N/m^2.

65. (a) Estimate the force with which a karate master strikes a board if the hand's speed at time of impact is 10.0 m/s, decreasing to 1.00 m/s during a 0.002 00-s time-of-contact with the board. The mass of his hand and arm is 1.00 kg. (b) Estimate the shear stress if this force is exerted on a 1.00-cm-thick pine board that is 10.0 cm wide. (c) If the maximum shear stress a pine board can support before breaking is 3.60×10^6 N/m^2, will the board break?

66. A bucket is made from thin sheet metal. The bottom and top of the bucket have radii of 25.0 cm and 35.0 cm, respectively. The bucket is 30.0 cm high and filled with water. Where is the center of gravity? (Ignore the weight of the bucket itself.)

67. **Review problem.** An aluminum wire is 0.850 m long and has a circular cross section of diameter 0.780 mm. Fixed at the top end, the wire supports a 1.20-kg object that swings in a horizontal circle. Determine the angular velocity required to produce a strain of 1.00×10^{-3}.

68. A bridge truss extends 200 m across a river (Fig. P12.68). The structure is free to slide horizontally to permit thermal expansion. The structural components are connected by pin joints, and the masses of the bars are small compared with the mass of a 1 360-kg car at the center. Calculate the force of tension or compression in each structural component.

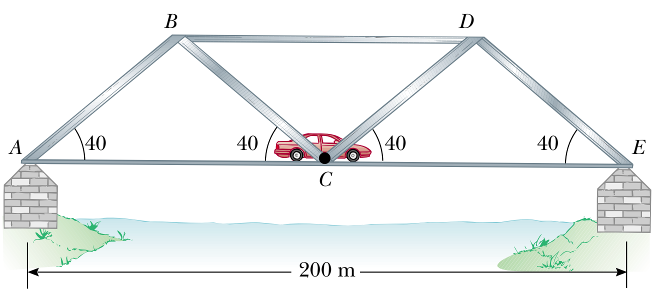


Figure P12.68

69. A bridge truss extends 100 m across a river (Fig. P12.69). The structure is free to slide horizontally to permit thermal expansion. The structural components are connected by pin joints, and the masses of the bars are small compared with the mass of a 1 500-kg car halfway between points *A* and *C*. Show that the weight of the car is in effect equally distributed between points *A* and *C*. Specify whether each structural component is under tension or compression and find the force in each.

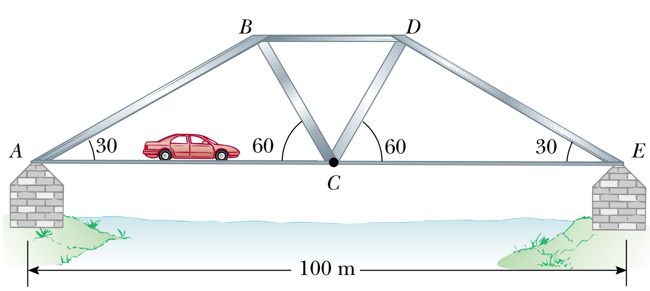


Figure P12.69

70. **Review problem.** A cue strikes a cue ball and delivers a horizontal impulse in such a way that the ball rolls without slipping as it starts to move. At what height above the ball's center (in terms of the radius of the ball) was the blow struck?

71. **Review problem.** A trailer with loaded weight F_g is being pulled by a vehicle with a force $\mathbf{P}$, as in Figure P12.71. The trailer is loaded such that its center of mass is located as shown. Neglect the force of rolling friction and let a represent the x component of the acceleration of the trailer. (a) Find the vertical component of $\mathbf{P}$ in terms of the given parameters. (b) If $a = 2.00\ \text{m/s}^2$ and $h = 1.50$ m, what must be the value of d in order that $P_y = 0$ (no vertical load on the vehicle)? (c) Find the values of P_x and P_y given that $F_g = 1\,500$ N, $d = 0.800$ m, $L = 3.00$ m, $h = 1.50$ m, and $a = -2.00\ \text{m/s}^2$.

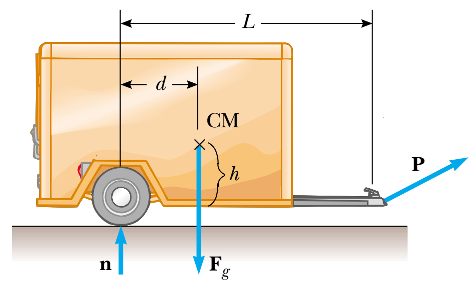


Figure P12.71

72. **Review problem.** A bicycle is traveling downhill at a high speed. Suddenly, the cyclist sees that a bridge ahead has collapsed, so she has to stop. What is the maximum magnitude of acceleration the bicycle can have if it is not to flip over its front wheel—in particular, if its rear wheel is not to leave the ground? The slope makes an angle of 20.0° with the horizontal. On level ground, the center of mass of the woman–bicycle system is at a point 1.05 m above the ground, 65.0 cm horizontally behind the axle of the front wheel, and 35.0 cm in front of the rear axle. Assume that the tires do not skid.

73. **Review problem.** A car moves with speed v on a horizontal circular track of radius R. A head-on view of the car is shown in Figure P12.73. The height of the car's center of mass above the ground is h, and the separation between its inner and outer wheels is d. The road is dry, and the car does not skid. Show that the maximum speed the car can

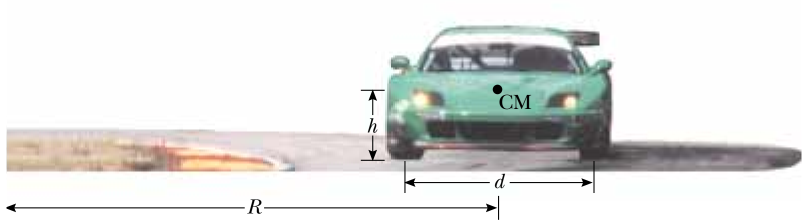


Figure P12.73

have without overturning is given by

$$v_{\max} = \sqrt{\frac{gRd}{2h}}$$

To reduce the risk of rollover, should one increase or decrease h? Should one increase or decrease the width d of the wheel base?

Answers to Quick Quizzes

12.1 (a). The unbalanced torques due to the forces in Figure 12.2 cause an angular acceleration even though the linear acceleration is zero.

12.2 (b). Notice that the lines of action of all the forces in Figure 12.3 intersect at a common point. Thus, the net torque about this point is zero. This zero value of the net torque is independent of the values of the forces. Because no force has a downward component, there is a net force and the object is not in force equilibrium.

12.3 (b). Both the object and the center of gravity of the meter stick are 25 cm from the pivot point. Thus, the meter stick and the object must have the same mass if the system is balanced.

12.4 (b). The friction force on the block as it slides along the surface is parallel to the lower surface and will cause the block to undergo a shear deformation.

12.5 (a). The stretching of the wire due to the increased tension is described by Young's modulus.

12.6 (c). The pressure of the atmosphere results in a force of uniform magnitude perpendicular at all points on the surface of the sphere.